The Semantics of Shape

Computational Methods for High-Level 3D Shape Analysis

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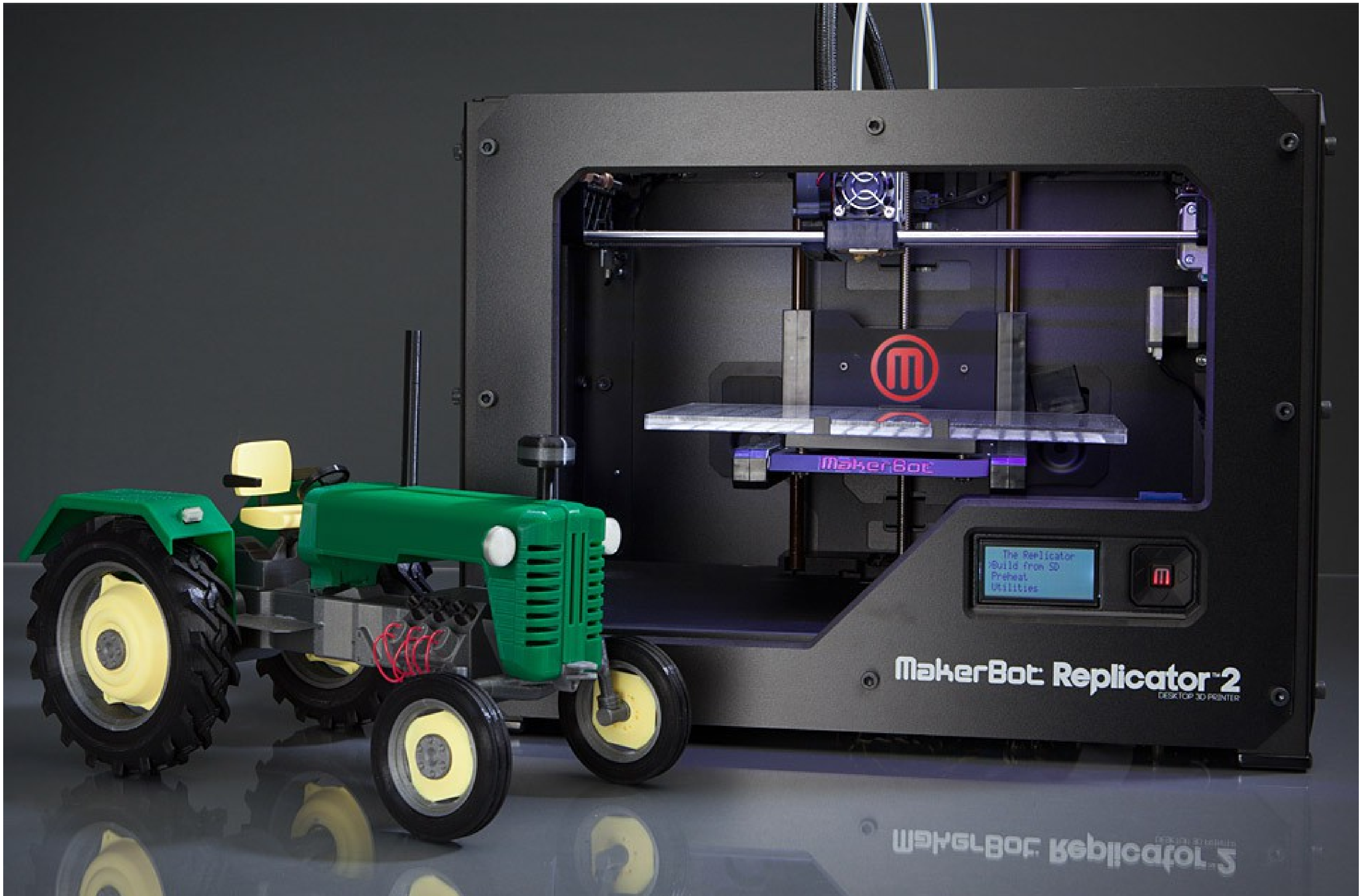
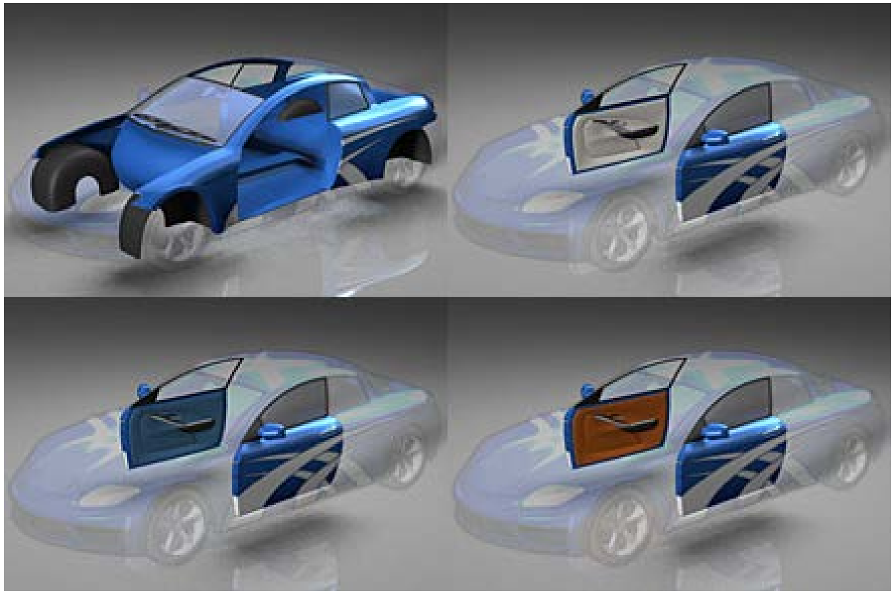
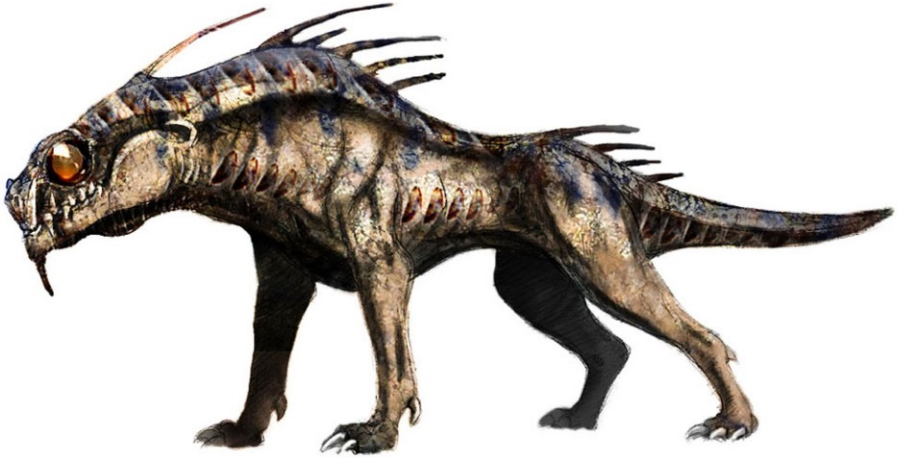
Shapes are everywhere!



MakerBot Industries

Shapes are everywhere!



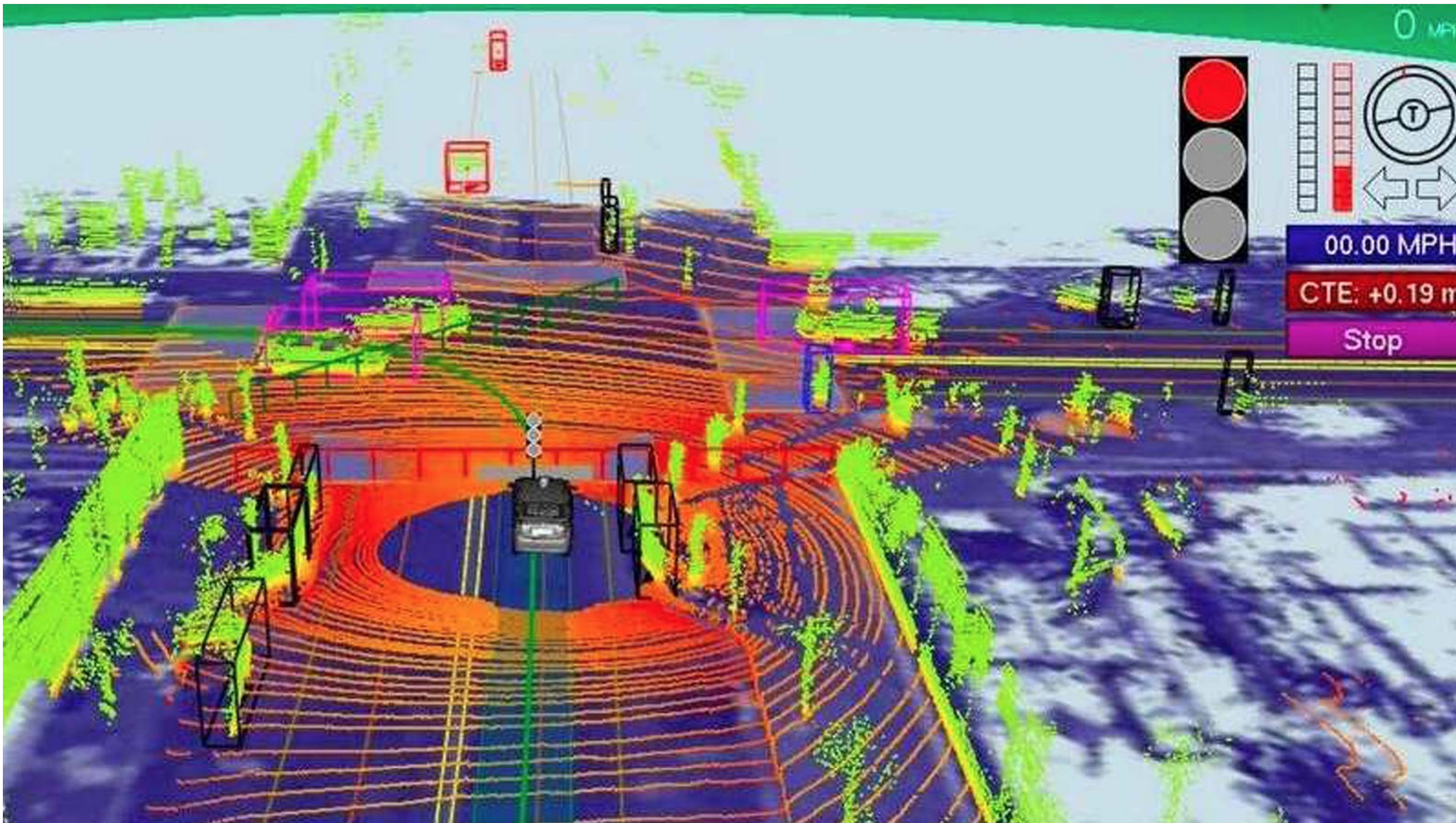


AFRL Discovery Lab/Aurora Flight Sciences

Shapes are everywhere!

Saxena Lab, Cornell/Stanford

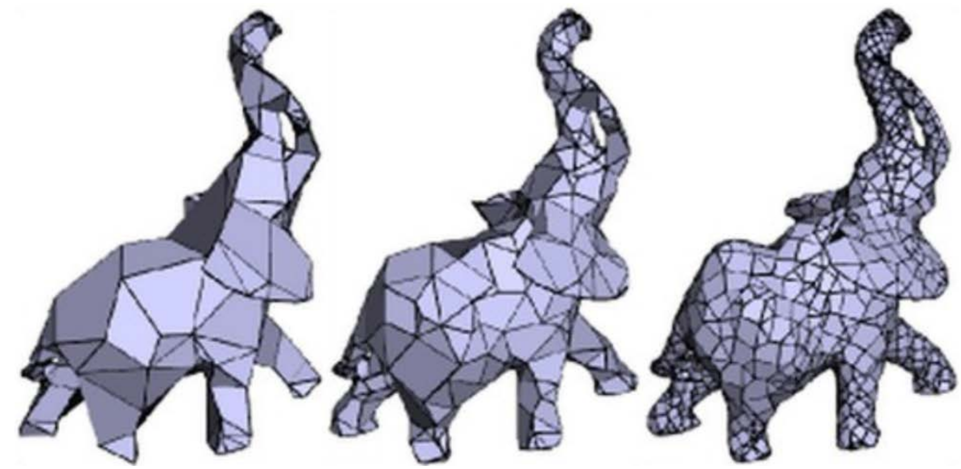
Shapes are everywhere!



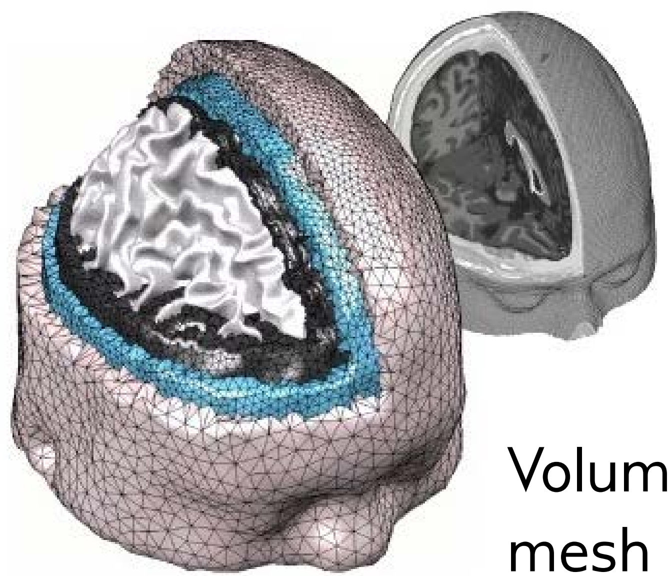
Google Inc.

Shape Representations

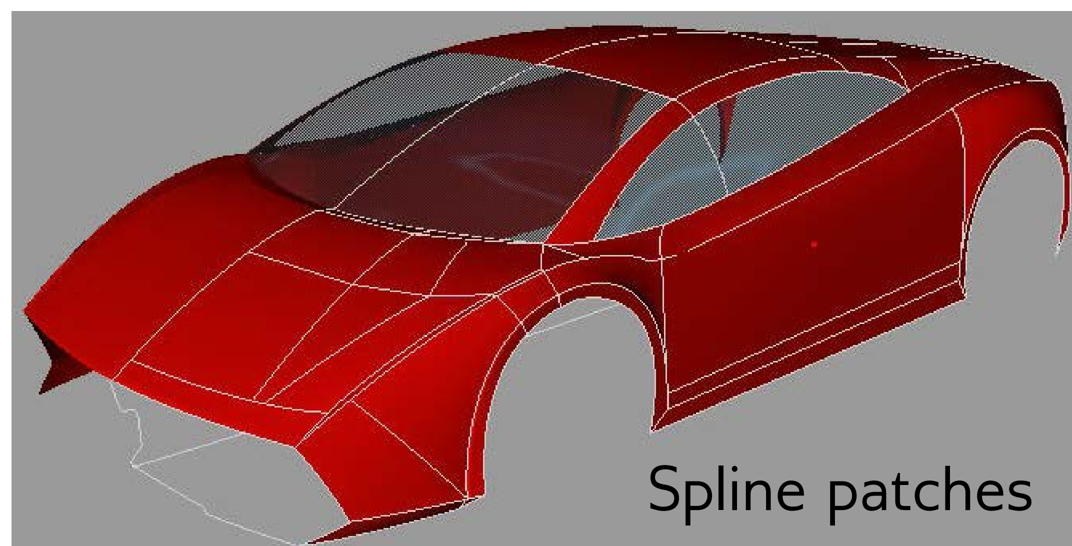
Point cloud



Polygon mesh

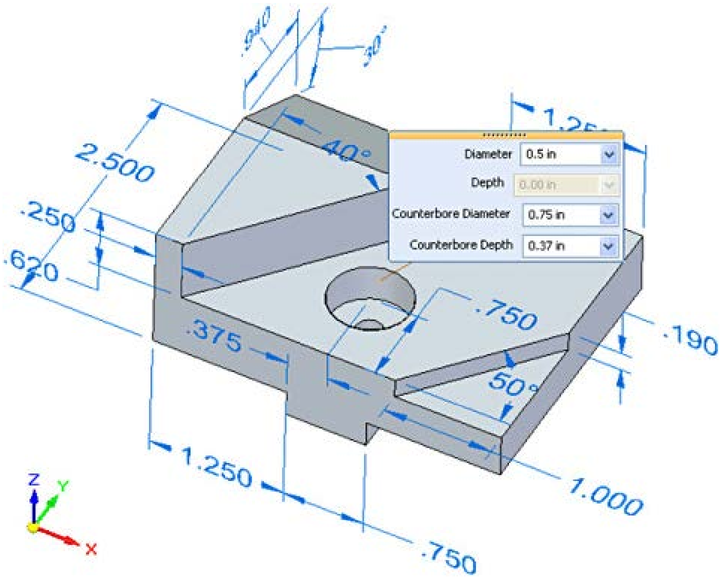


Volumetric mesh

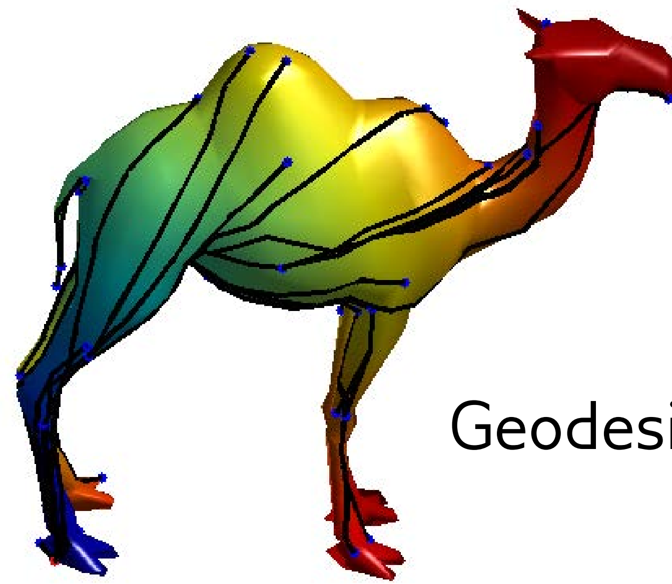


Spline patches

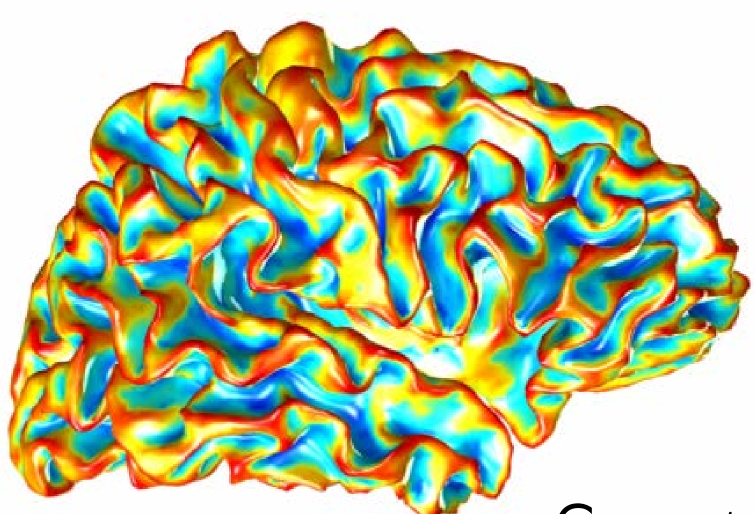
"Low-Level" Geometric Analysis



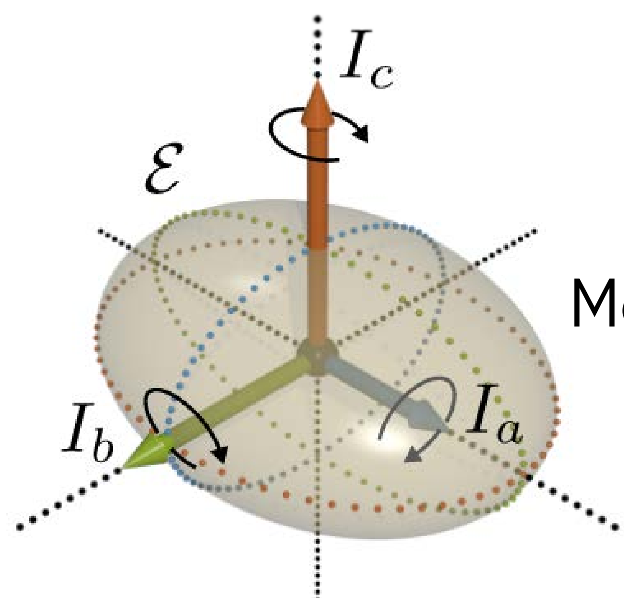
Dimensions



Geodesics

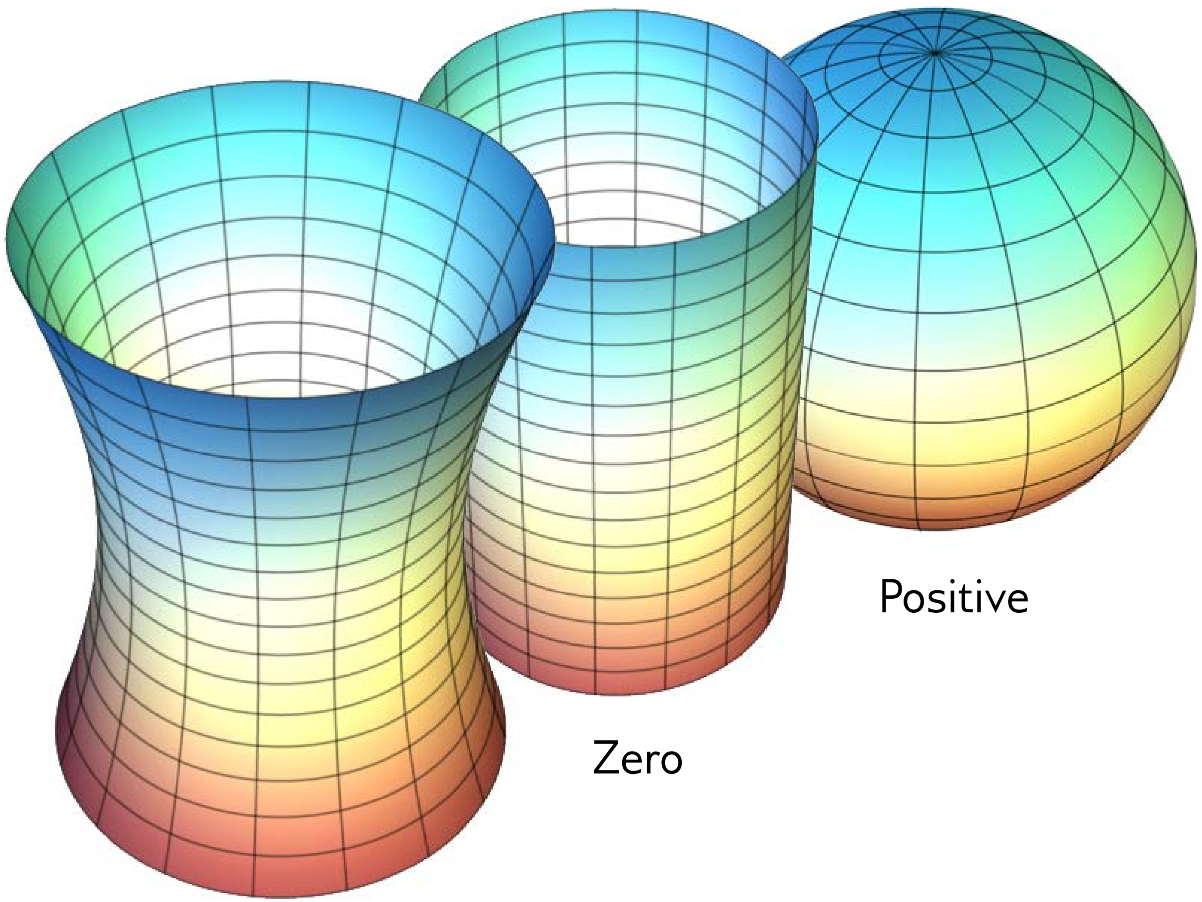


Curvature



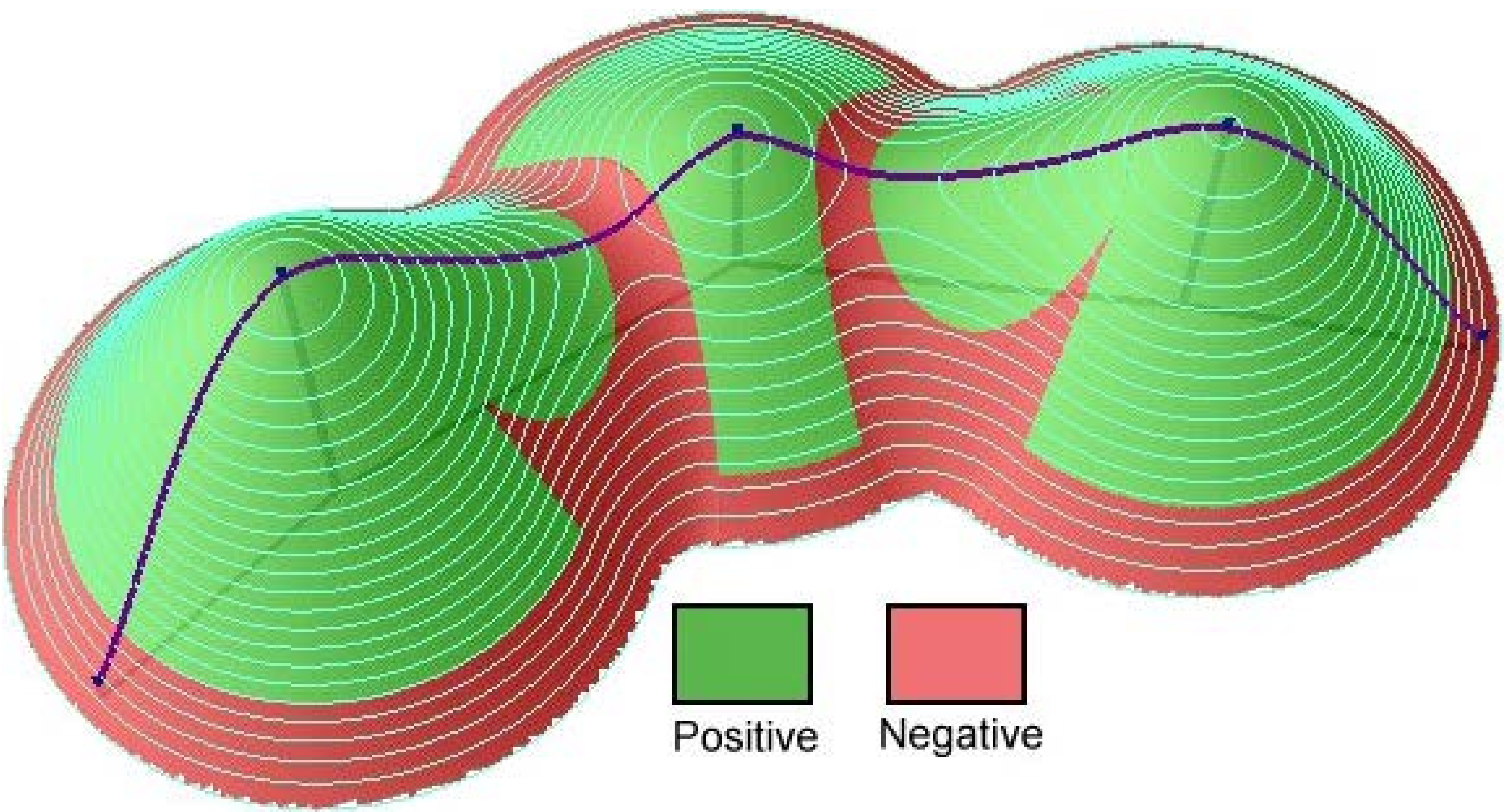
Moments

Gaussian Curvature



Jhausauer@wikipedia

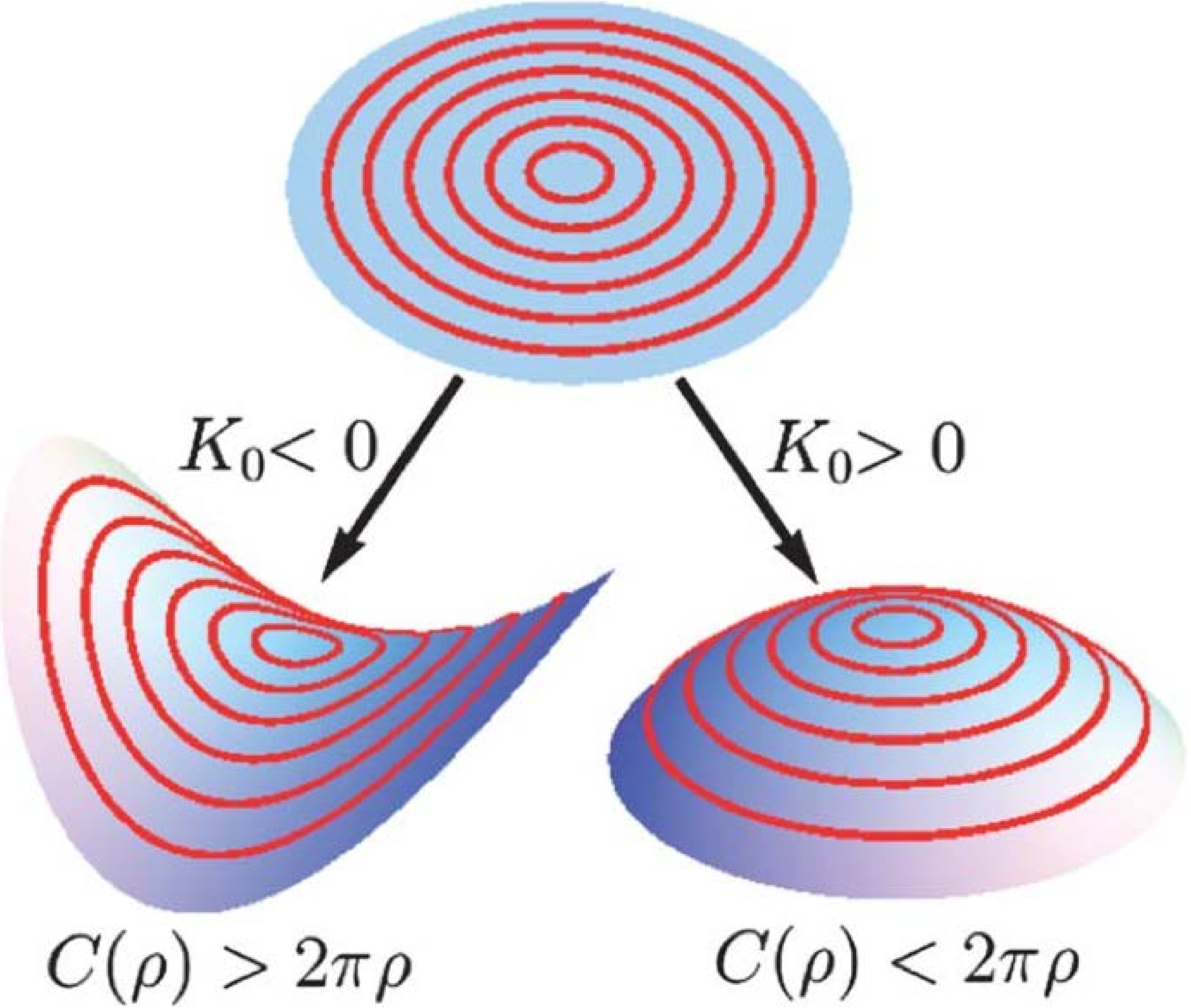
Gaussian Curvature



Can a 2D ant on a 2D surface tell if it lives in a space of positive, negative or zero curvature?

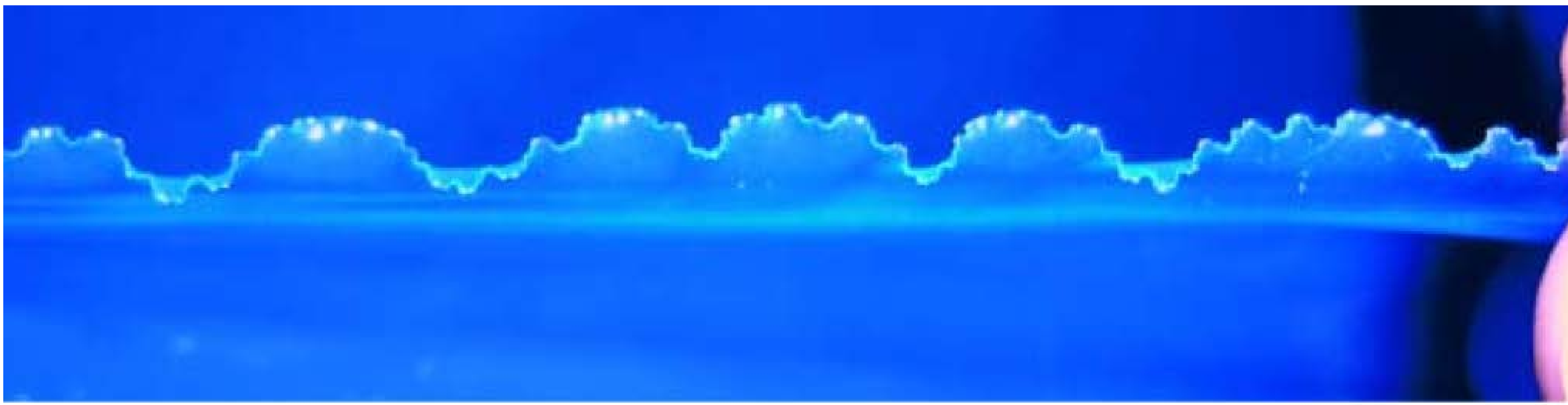
Can a person, in 3D?

Yes, by measuring distances!



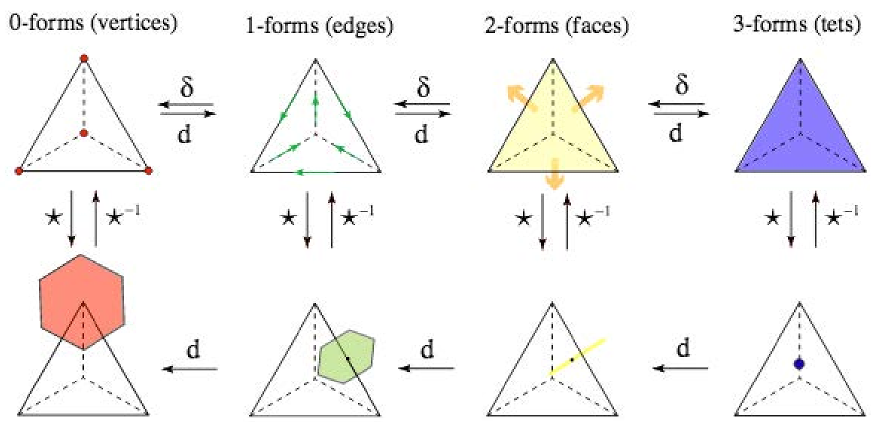
Gemmer and Venkataramani, 2013

Sharon et al. 2004

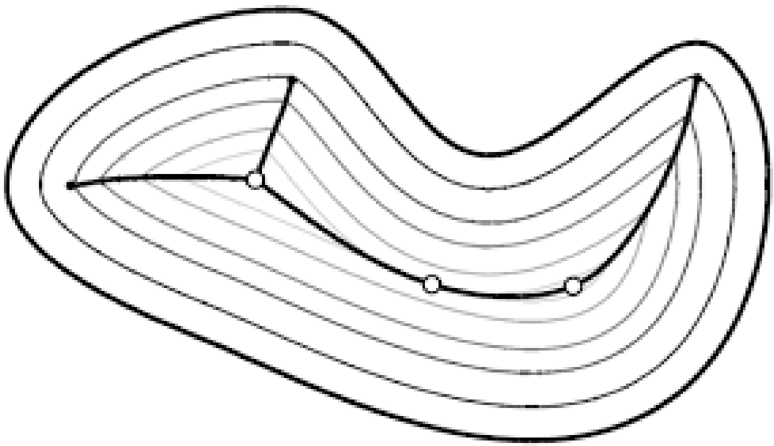


Sharon et al. 2004

"Low-Level" Geometric Analysis



Discrete Differential Geometry



Medial Axis Transform

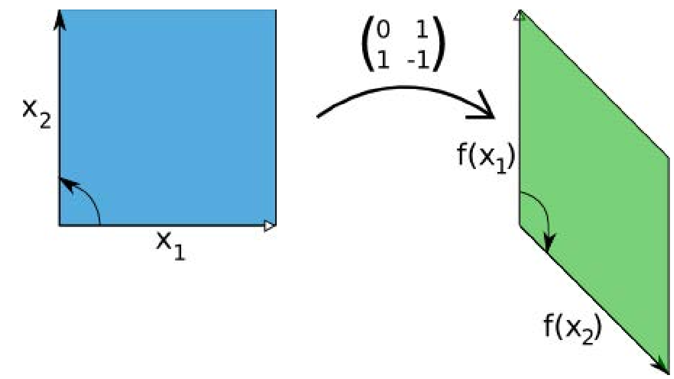
Spectral Decomposition

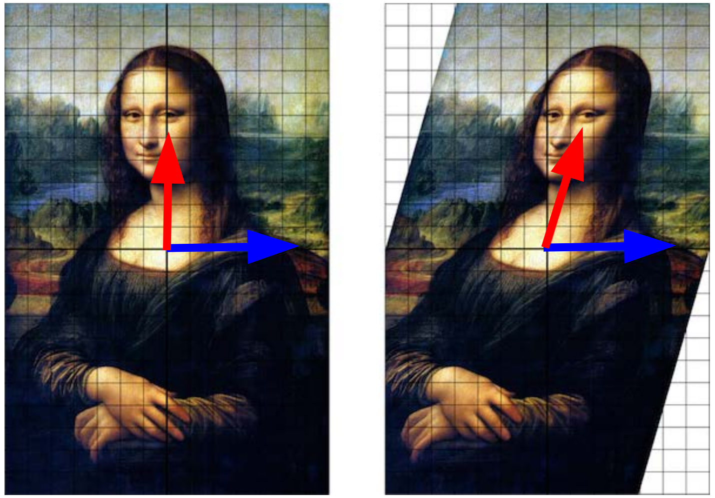
Matrices as transformations

- Let A be an $n \times n$ matrix
  - It can be thought of as a function that maps a vector $\mathbf{x} \in \mathbb{R}^n$ to a vector $A\mathbf{x} \in \mathbb{R}^n$
- A is a **linear transformation**
  - f is linear if $f(a+b) = f(a) + f(b)$
- An **eigenvalue** of A is a scalar λ such that

$$A\mathbf{x} = \lambda\mathbf{x}$$

  where $\mathbf{x}$ is some n-D vector
- $\mathbf{x}$ is the corresponding **eigenvector**



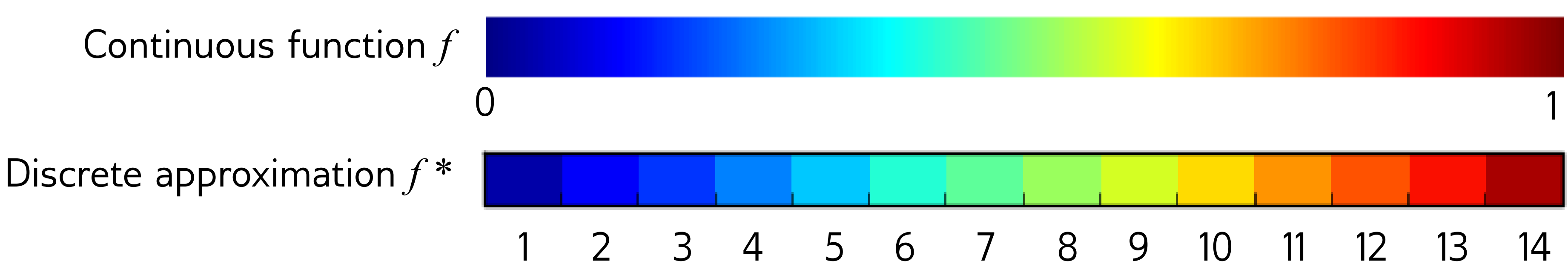


Blue arrow is eigenvector of shear transform, red is not

StackExchange, Wikipedia

Functions as vectors

- Functions from A to B form a vector space: we can think of functions as "vectors"
  - E.g. we can commutatively add two functions: $f + g = g + f$
  - Or distribute multiplication with a scalar: $s(f + g) = sf + sg$
- A function f can be **discretized** to an n-D vector of sampled values: $[f(x_1), f(x_2), \ldots, f(x_n)]$



Linear operators

- An **operator** T is a mapping from a vector space U to another vector space V
  - T is a **linear operator** if $T(a+b) = T(a) + T(b)$
- The set of functions F from domain A to codomain B is a vector space
  - So we can have operators T that map from one function space F to another function space G
  - Note that T maps functions to functions!
- The differentials $\frac{d}{dx}, \frac{d^2}{dx^2}, \frac{d^3}{dx^3}$ etc are linear operators
  - They map functions to their derivatives

Eigenfunctions of operators

- An **eigenvalue** of a linear operator T that maps a vector space to itself is a scalar λ s.t.

$$T(\mathbf{x}) = \lambda \mathbf{x}$$

  and $\mathbf{x}$ is the corresponding **eigenvector**

- If T maps functions to functions, then we call $\mathbf{x}$ an **eigenfunction**: $T(f) = \lambda f$

Discrete Linear Operators

- **Theorem:** Any linear operator between finite-dimensional vector spaces can be represented by a matrix
  - Let's say we have a set of functions F from A to B
  - The discrete versions of the functions form a finite-dimensional vector space F^* equivalent to $\mathbb{R}^n$
    - Each function is sampled at the same finite set of points
  - Let T be a linear operator from F to itself
  - ... and T^* be a "discrete version" of T acting on F^*
  - Then T^* can be represented by a $n \times n$ matrix (cf. theorem)

Example: Discrete Derivative

Continuous

- Function: f
- Operator: $\frac{d}{dx}$
- Applying operator:

$$\frac{df}{dx} = f\,'$$

Discrete

- Vector: $\mathbf{f} = [\,f(x_1), f(x_2) \ \ldots \ f(x_n)]$
- Matrix:

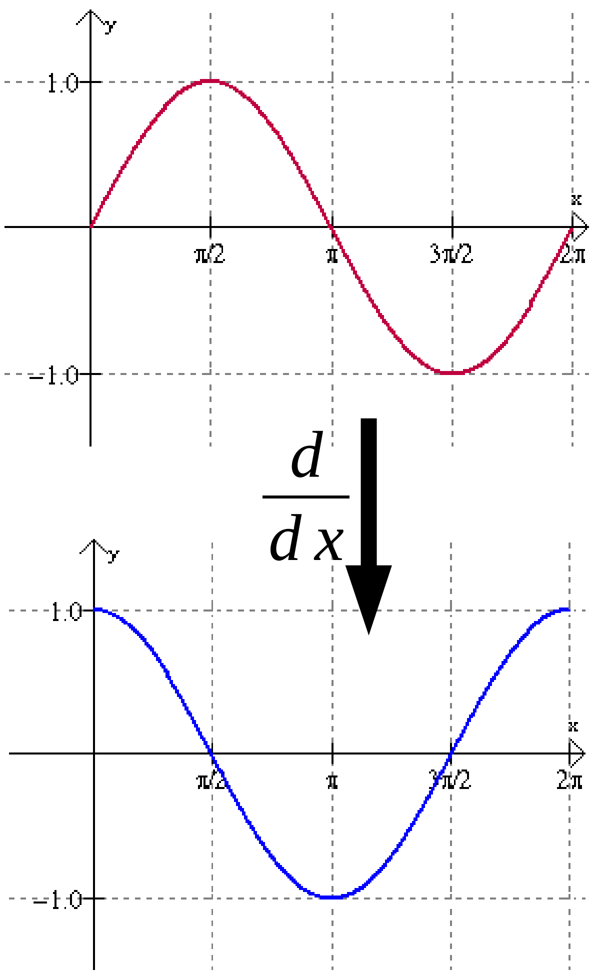
$$A = \frac{1}{h}\begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ & 0 & -1 & \cdots & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & & & -1 & 1 \\ 0 & 0 & \cdots & & 0 & -1 \end{bmatrix}$$

- Applying matrix:

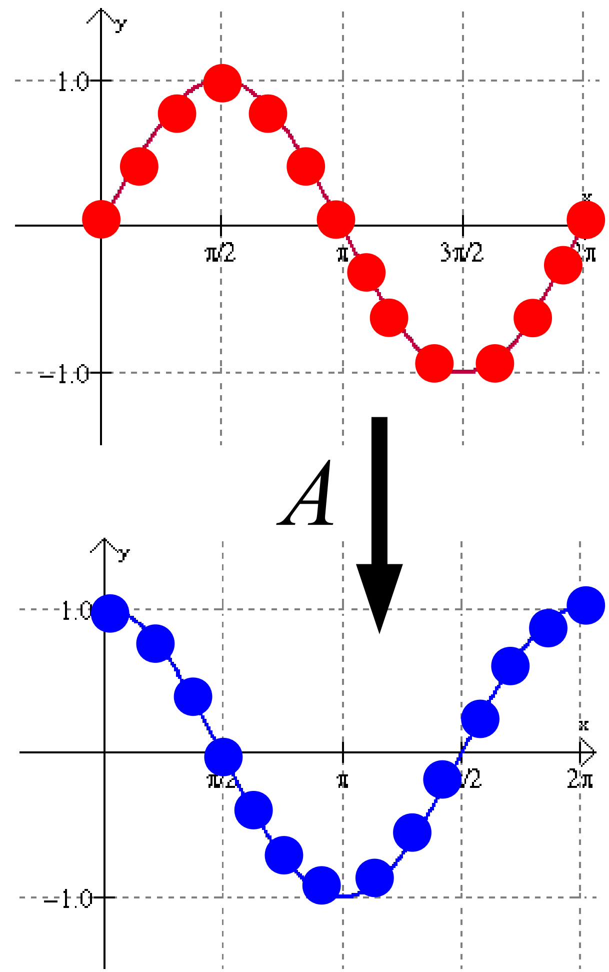
$$A\mathbf{f} = \mathbf{f}\,'$$

Example: Discrete Derivative

Continuous



Discrete



Example: Discrete 2nd Derivative

Continuous

- Function: f
- Operator: $\dfrac{d^2}{d\,x^2}$
- Applying operator:

$$\frac{d^2 f}{d\,x^2} = f''$$

Discrete

- Vector: $\mathbf{f} = [\,f(x_1), f(x_2)\ \ldots\ f(x_n)]$
- Matrix:

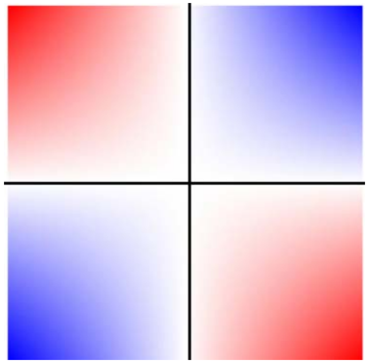
$$L = \frac{1}{h^2}\begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ & 1 & -2 & \cdots & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & & & -2 & 1 \\ 0 & 0 & \cdots & & 1 & -2 \end{bmatrix}$$

- Applying matrix:

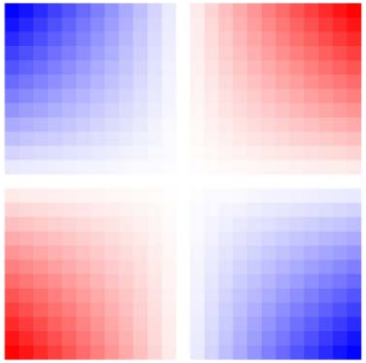
$$L\mathbf{f} = \mathbf{f}''$$

Operators in higher dimensions

- The underlying function space can have a higher-dimensional domain



Continuous function



Discrete approximation

$$\begin{bmatrix} -4 & 1 & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & -4 & 1 & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & -4 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & -4 & 1 & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 & -4 & 1 & \cdot & 1 & \cdot \\ \cdot & \cdot & 1 & \cdot & 1 & -4 & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & -4 & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & 1 & -4 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & 1 & -4 \end{bmatrix}$$

2D discrete Laplace operator

Interpreting eigenfunctions

- Eigenvalues of a linear operator form its **spectrum**
- The eigenfunctions are unchanged (except for scaling) when transformed by the operator
  - Think of them as standing waves on the domain
- E.g.

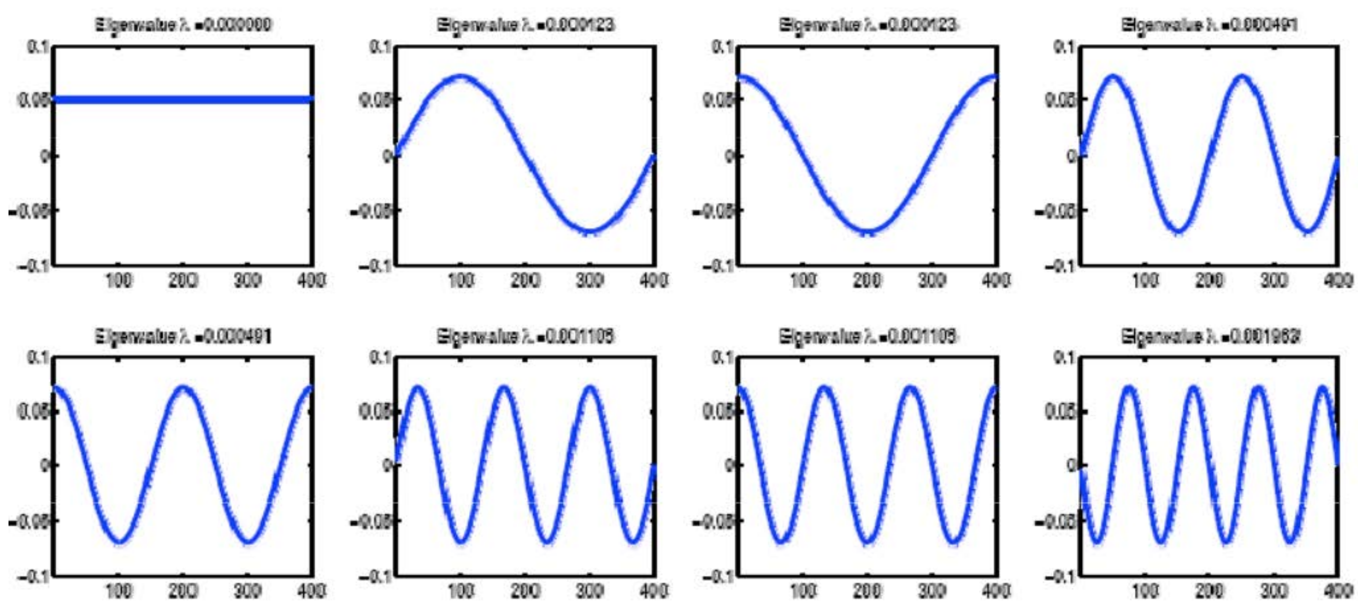
$$\frac{d^2 \sin(nx)}{dx^2} = -n^2 \times \sin(nx)$$

$$\frac{d^2 \cos(nx)}{dx^2} = -n^2 \times \cos(nx)$$

$$\frac{d^2 e^{\lambda x}}{dx^2} = \lambda^2 \times e^{\lambda x}$$

Interpreting eigenfunctions

- The eigenfunctions of the operator form a basis for the function space
  - E.g. the sinusoidal eigenfunctions of $\frac{d^2}{dx^2}$ form the Fourier basis



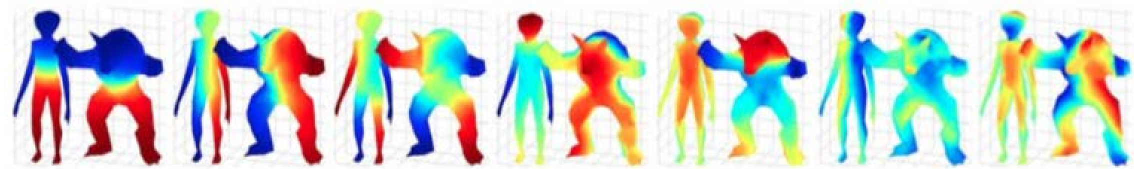
The first 8 sinusoidal eigenfunctions of the second derivative operator.
The eigenvalues are the negative squared frequencies.

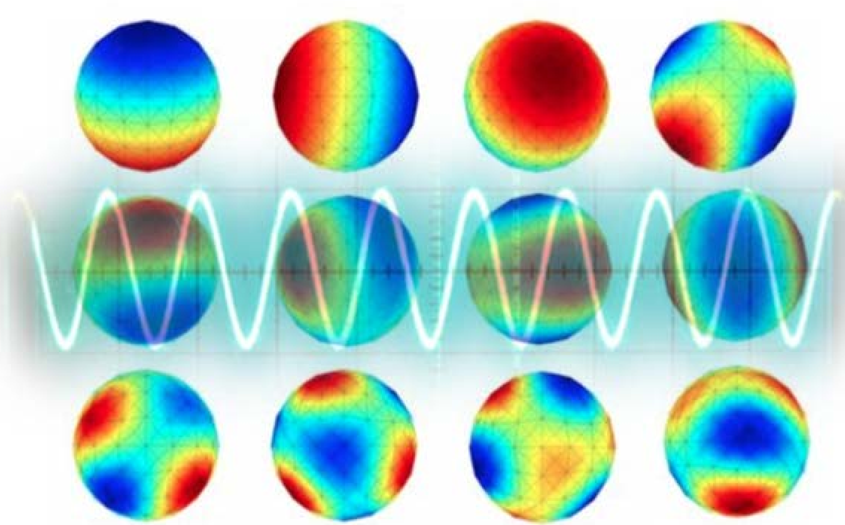
Operators on manifolds

- We can define a function on a manifold curve/surface!
  - E.g. the coordinate function: gives the (X, Y, Z) position of a point on the surface
- A common operator is the Laplace-Beltrami operator
  - Its eigenfunctions define a basis for functions over the surface

Eigenfunctions of Laplace-Beltrami

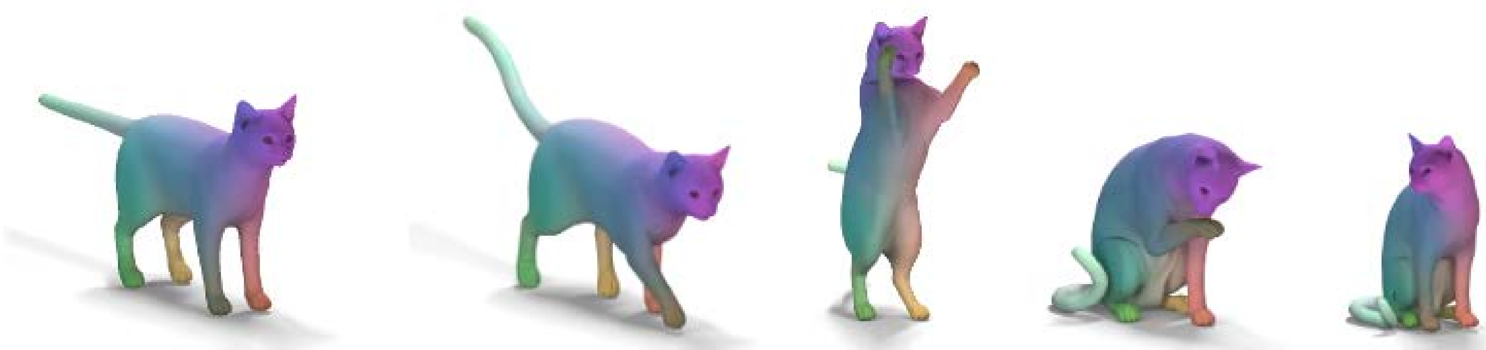
- *Intrinsic* basis for functions over surface
  - Doesn't change under isometry
- We can discretize it as usual: the function is defined at a fixed set of sample points on the shape





Eigenfunctions of Laplace-Beltrami

- The spectrum of the L-B operator characterizes the intrinsic geometry of the shape
- Two shapes related by isometry have the same Laplace-Beltrami spectrum



Expressing a function with eigenfunctions

- **Continuous:**

$$f(p) = w_1\,\varphi_1(p) + w_2\,\varphi_2(p) + \ldots + w_n\varphi_n(p)$$

- **Discrete:**

$$X = \sum_{i=1}^{n} \mathbf{e}_i \tilde{x}_i = \begin{bmatrix} E_{11} \\ E_{21} \\ \vdots \\ E_{n1} \end{bmatrix} \tilde{x}_1 + \ldots + \begin{bmatrix} E_{1n} \\ E_{2n} \\ \vdots \\ E_{nn} \end{bmatrix} \tilde{x}_n = \begin{bmatrix} E_{11} & \ldots & E_{1n} \\ E_{21} & \ldots & E_{2n} \\ \vdots & \vdots & \vdots \\ E_{n1} & \ldots & E_{nn} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_n \end{bmatrix} = E\tilde{X}$$

$$\tilde{X} = E^{\mathrm{T}} X$$

$$\boxed{\tilde{x}_i = \mathbf{e}_i^{\mathrm{T}} \cdot X.}$$

Projection of *X* along eigenvector

The spectral transform

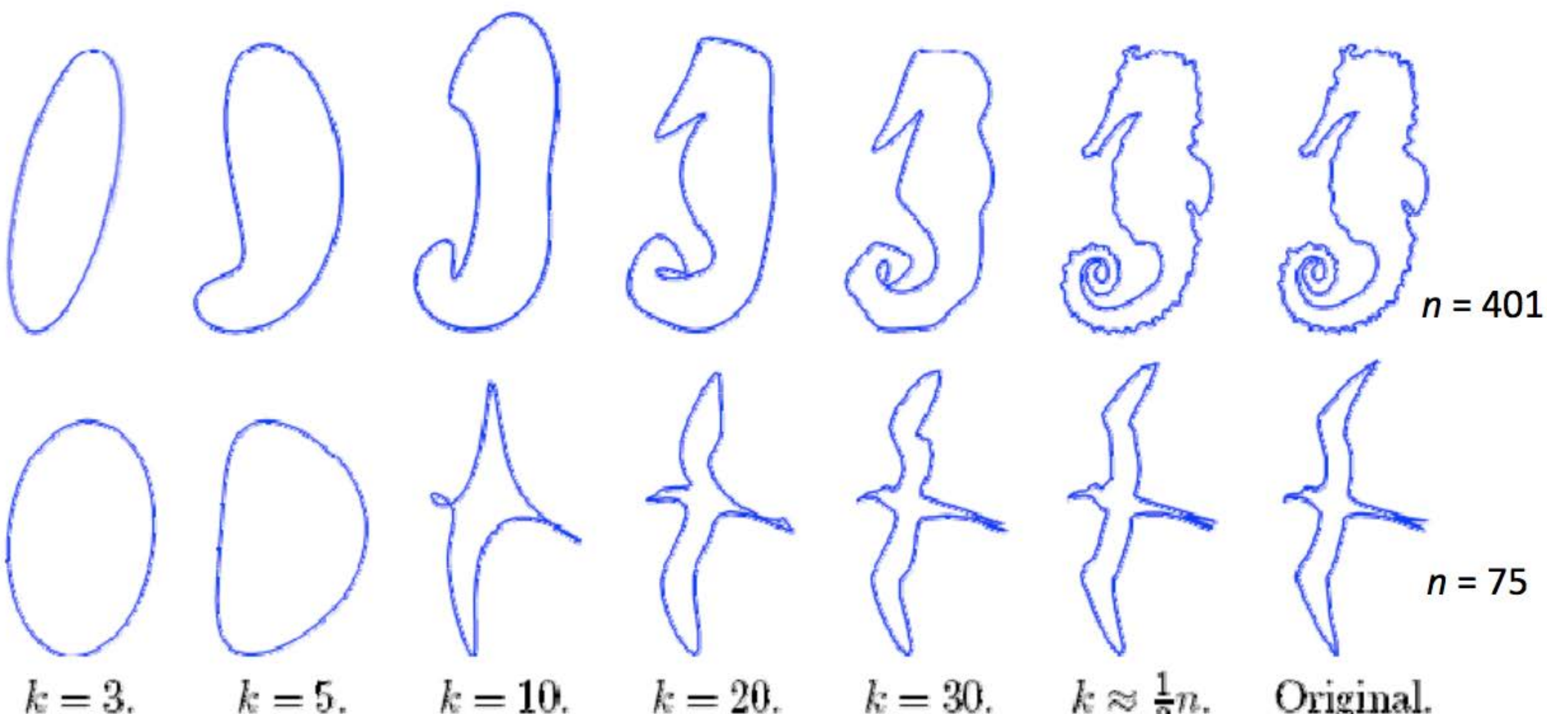
$$X \longrightarrow \tilde{X}$$

Spatial domain

Spectral domain

Reconstruction in 2D

- More accuracy with more eigenfunctions
- Function is the coordinate function
    - We're reconstructing the extrinsic shape of the object



Reconstruction in 3D

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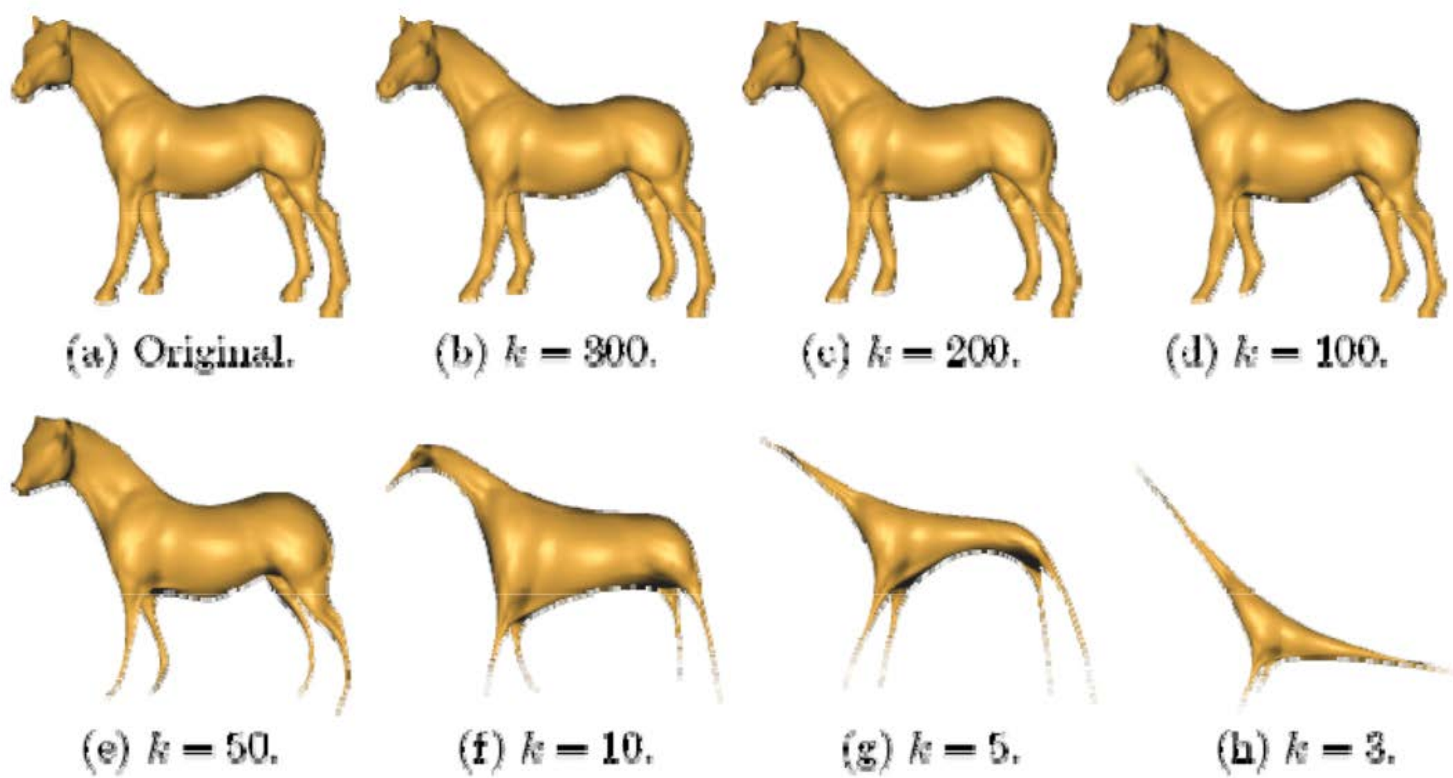
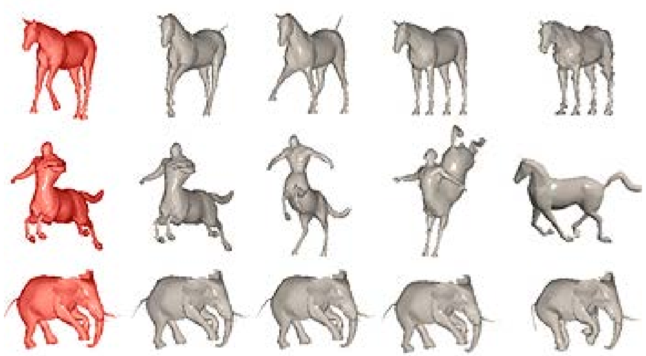


(a) Original. (b) $k = 300$. (c) $k = 200$. (d) $k = 100$.

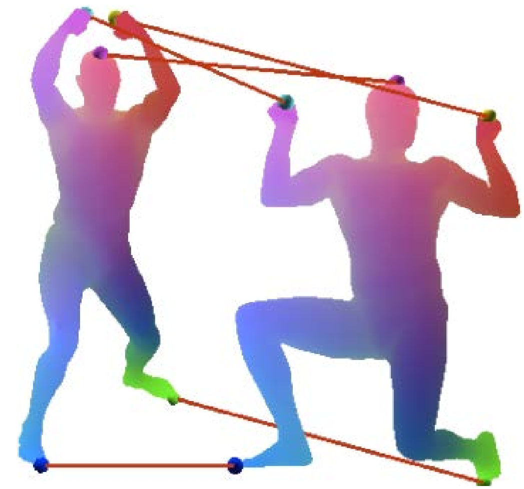
(e) $k = 50$. (f) $k = 10$. (g) $k = 5$. (h) $k = 3$.

"Mid-Level" Geometric Analysis

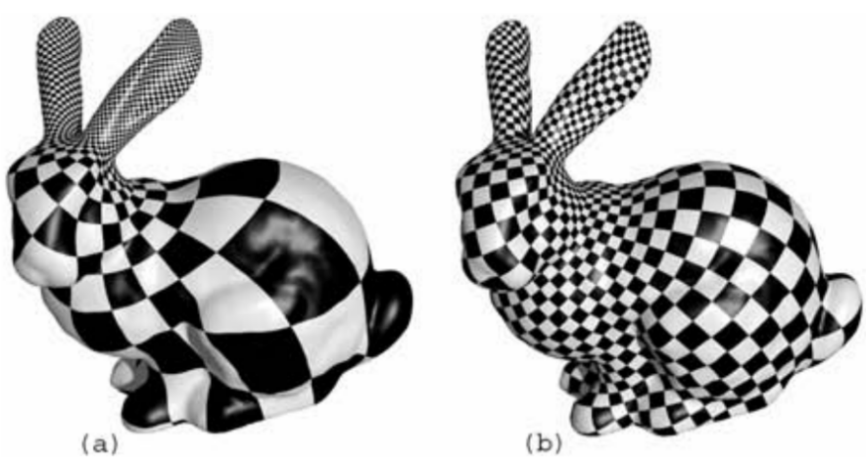


Retrieval

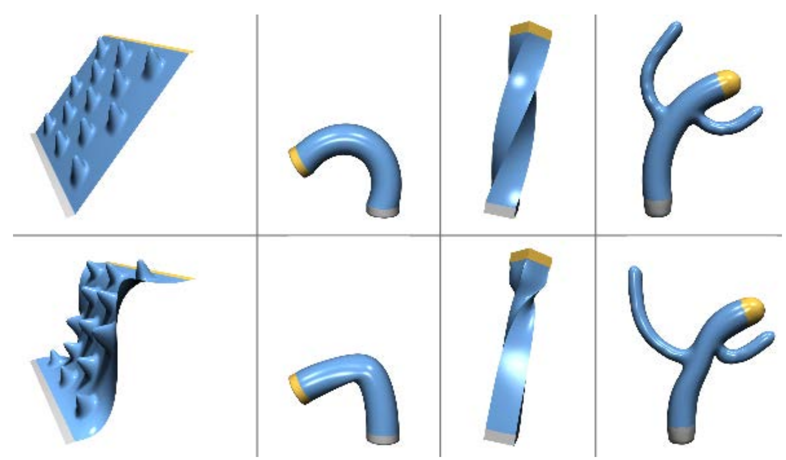
Segmentation



Correspondences



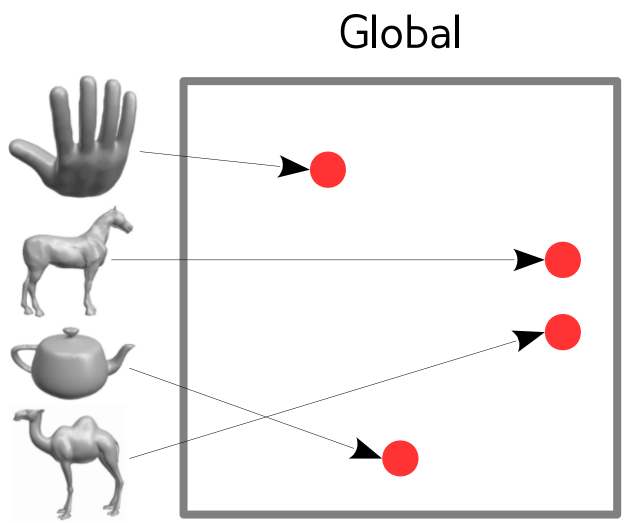
Parametrization

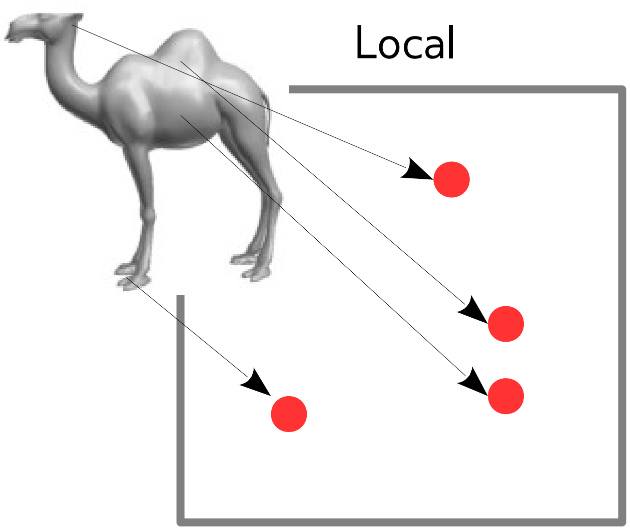


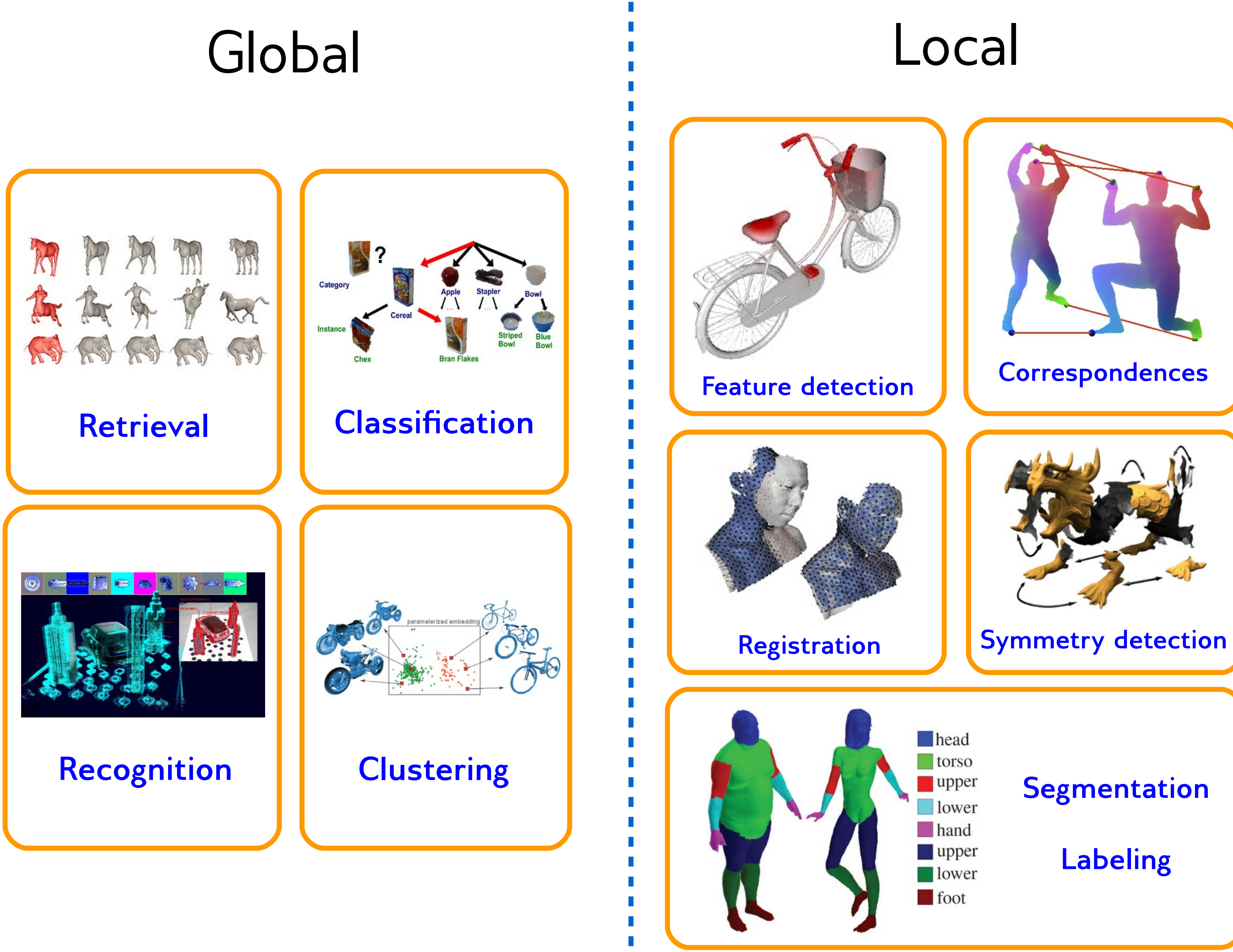
Deformation

Shape Descriptors

- A **shape descriptor** is a set of numbers that describes a shape in a way that is
  - **Concise**
  - **Quick to compute**
  - **Efficient to compare**
  - **Discriminative**
- **Global descriptors** describe whole objects
- **Local descriptors** describe (neighborhoods around) points
- Typically, the descriptors form a vector space with a meaningful distance metric



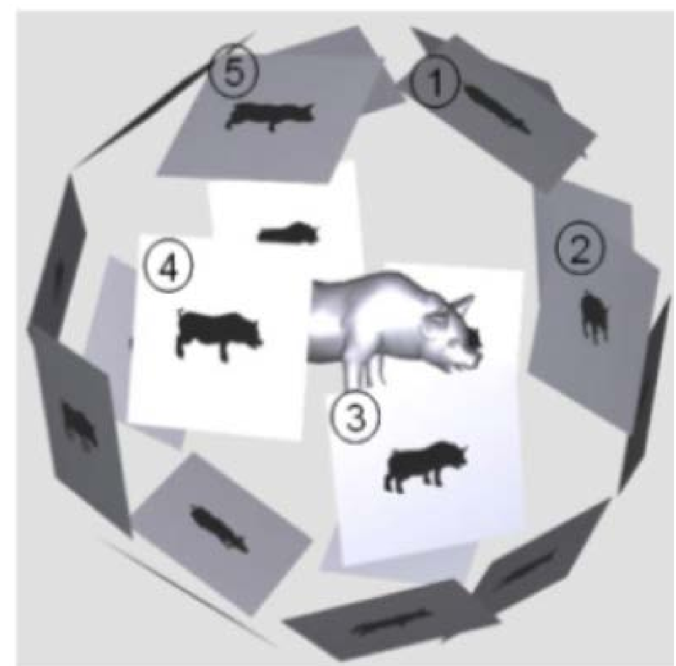




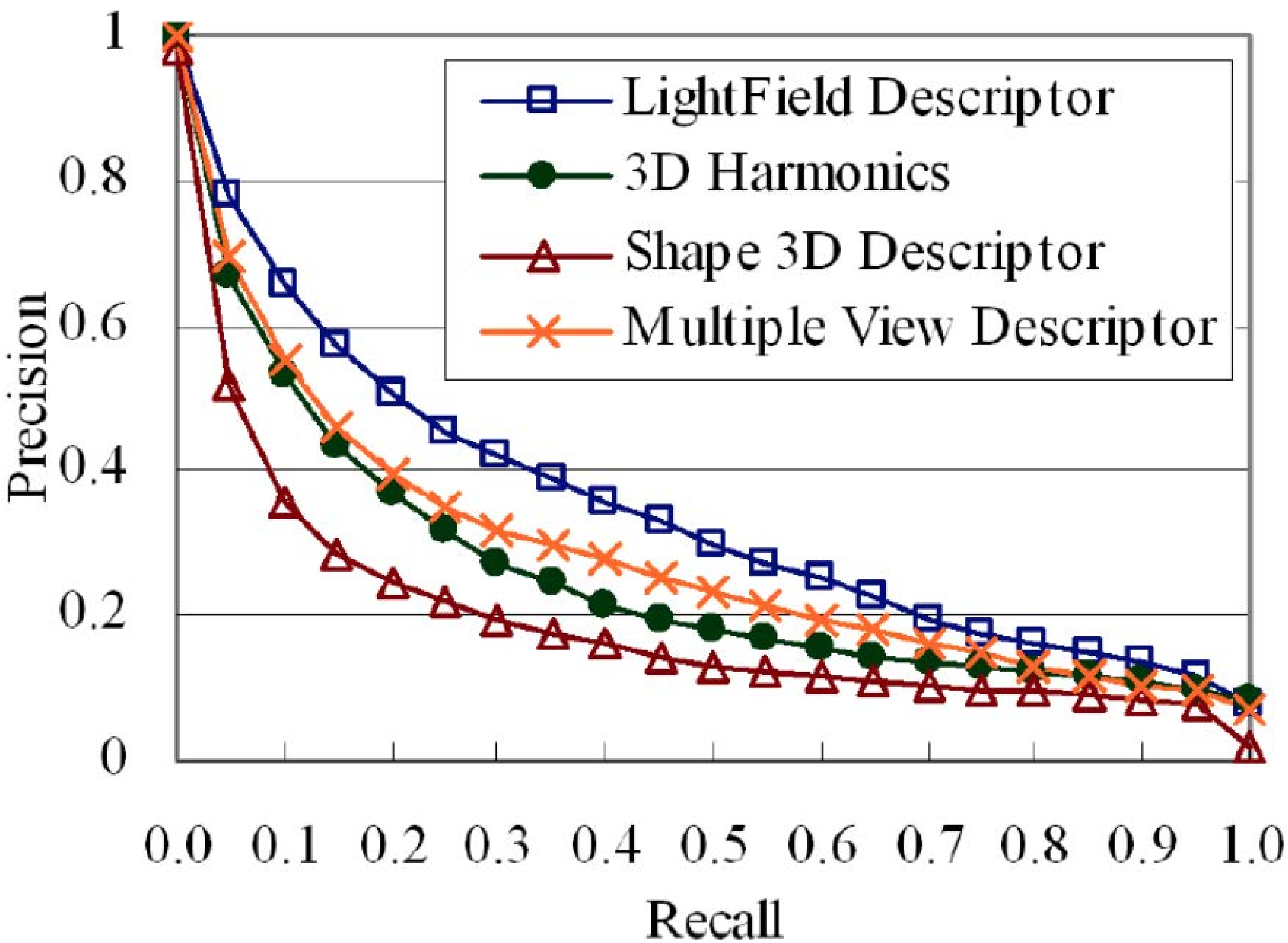
Global
Local
Category
Instance
Cereal
Apple
Stapler
Bowl
Chex
Bran Flakes
Striped Bowl
Blue Bowl
Retrieval
Classification
parameterized embedding
Recognition
Clustering
Feature detection
Correspondences
Registration
Symmetry detection
head
torso
upper
lower
hand
upper
lower
foot
Segmentation
Labeling

LFD: a classic global descriptor

- The **Light Field Descriptor** (LFD) of a 3D shape is a set of 2D images of it, taken with a camera array
  - E.g. 20 cameras positioned at the vertices of a regular dodecahedron
  - Images rendered as silhouettes, so 10 unique views (say from a hemisphere)
  - Instead of the actual images, store their **Zernike Moments** and **Fourier Descriptors**
  - Compare shapes over all possible relative rotations of image clouds



Retrieval Results



3D Harmonics: spectral signature of the shape

Shape 3D Descriptor: curvature histograms

Multiple View Descriptor: align shapes using PCA, compare views along principal axes

Test database: 1833 shapes, with 549 shapes classified into 47 functional categories, the remaining shapes classified as "miscellaneous"

What if we use better image descriptors?

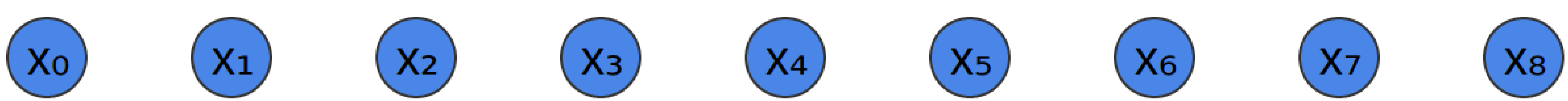
- ZMD/FD are ok, but hardly the state of the art in modern computer vision (circa 2016)
- Convolutional Neural Nets (CNNs) have revolutionized image recognition tasks

Model	**Top-1**	**Top-5**
Sparse coding [2]	*47.1%*	*28.2%*
SIFT + FVs [24]	*45.7%*	*25.7%*
CNN	**37.5%**	**17.0%**

In 2012, the error rate in the ImageNet visual recognition challenge was halved by a deep CNN (gains are typically incremental). There are 1000 categories: the baseline of random guessing would have a 99.9% error.

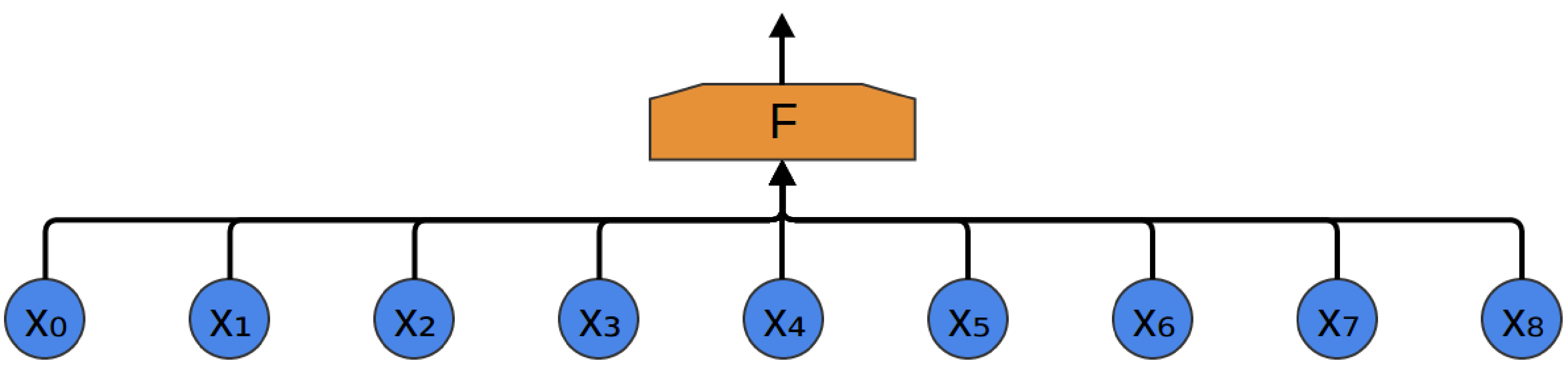
What is a Convolutional Neural Network?

- Imagine we have a set of N samples from some signal
- We want to produce a prediction, e.g. whether the signal represents a human voice, or a picture of a cat, or a depth image of a building



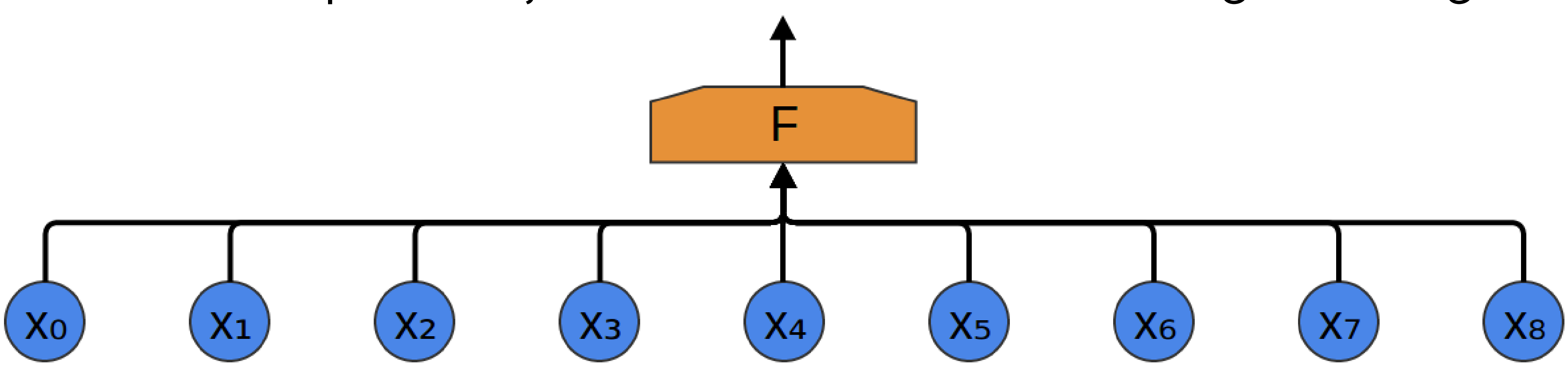
What is a Convolutional Neural Network?

- We can compute the probability as a function F of these values
    - In a **fully-connected** network, the function takes in all the inputs at once, e.g. as $g(\mathbf{w}\cdot\mathbf{x})$, where $\mathbf{w}$ is a weight vector and g is some nonlinear transformation such as a sigmoid function



What is a Convolutional Neural Network?

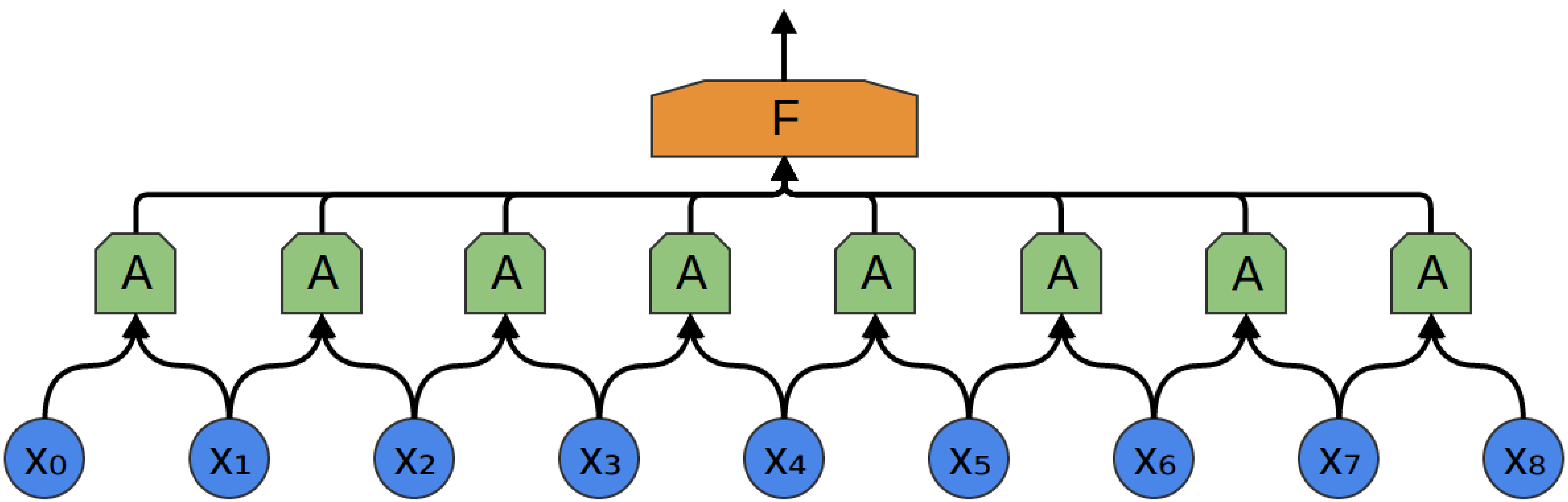
- Fully-connected networks have some drawbacks
  - The function is **very high-dimensional** (all inputs processed at once)
  - **No complex relationships** between inputs are modeled (just a dot product)
  - Local information is **not captured in a "translation-invariant" way** (a feature of the signal at the left end of the sequence must be learned independently of the same feature occurring at the right end)



Christopher Olah

What is a Convolutional Neural Network?

- **Solution:** a **convolutional layer**
- A filter (again, a dot product followed by a nonlinear transformation) is applied on local neighborhoods of the signal

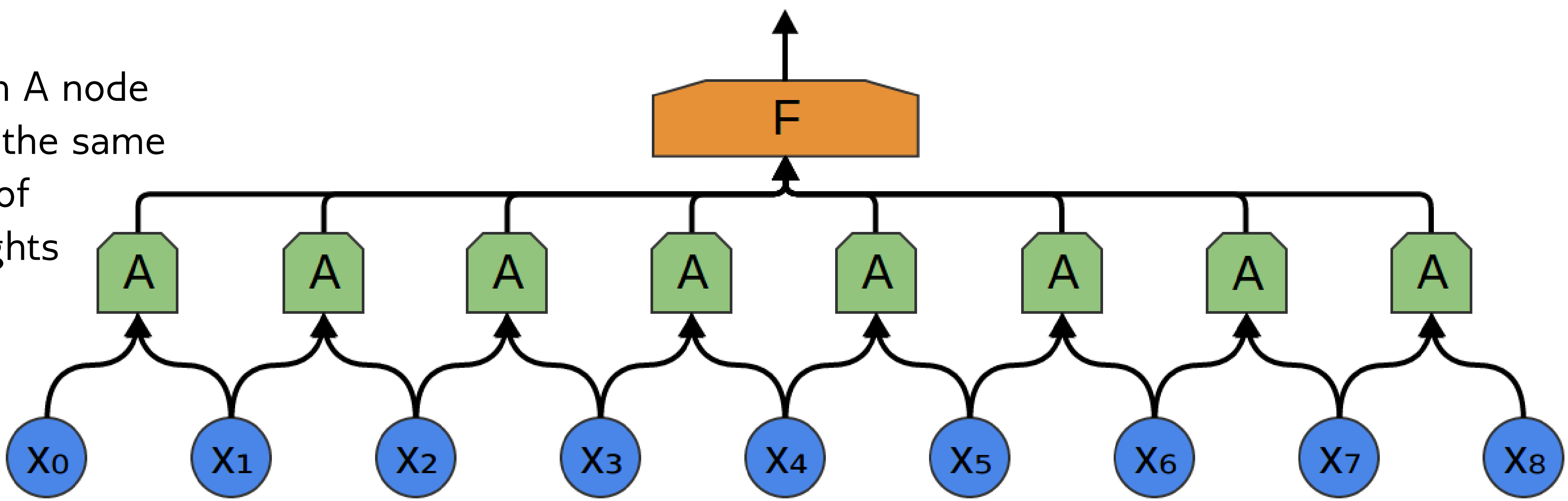


Christopher Olah

What is a Convolutional Neural Network?

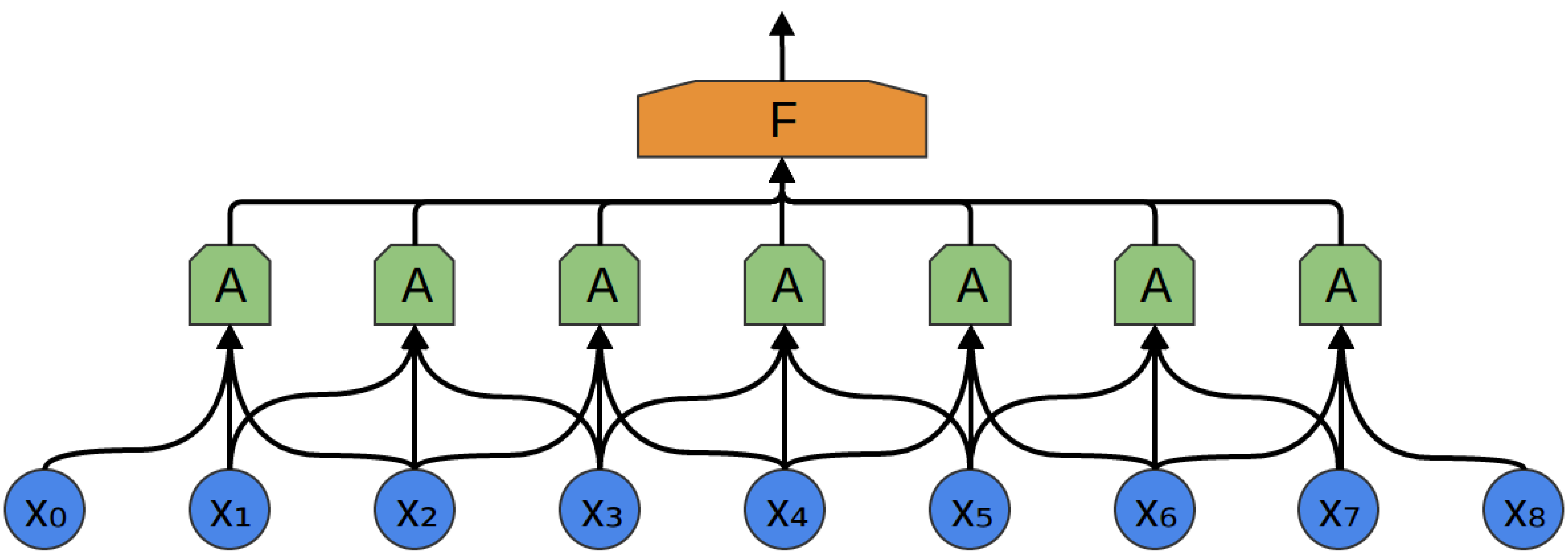
- All filters **share the same weights**!
  - Dramatically reduces number of parameters of the network
- The final output is a function of the filter responses

Each A node has the same set of weights



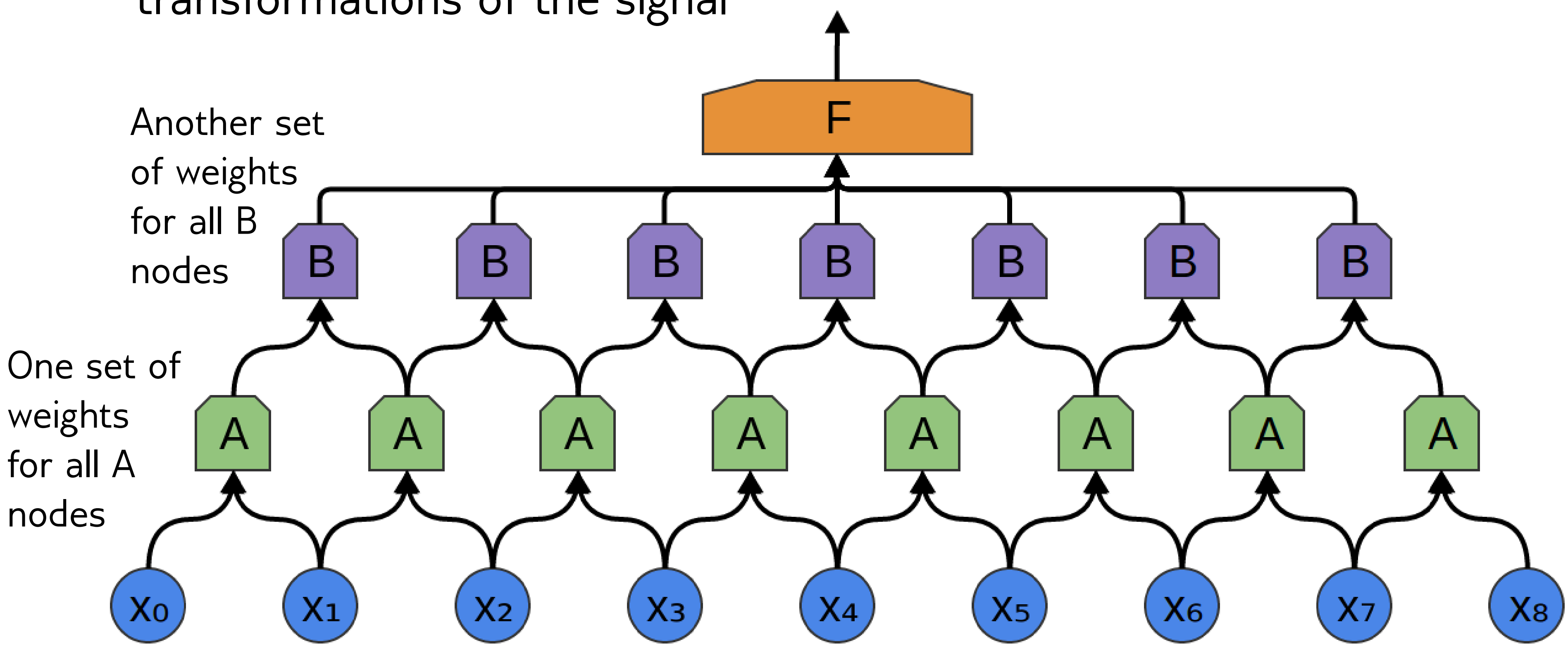
What is a Convolutional Neural Network?

- We can make the neighborhoods larger, to capture broader local features



What is a Convolutional Neural Network?

- Convolutional layers are **composable**: they can be stacked with each layer providing inputs for the next layer
  - Higher layers can capture more abstract features since they effectively cover larger neighborhoods, and combine multiple different nonlinear transformations of the signal

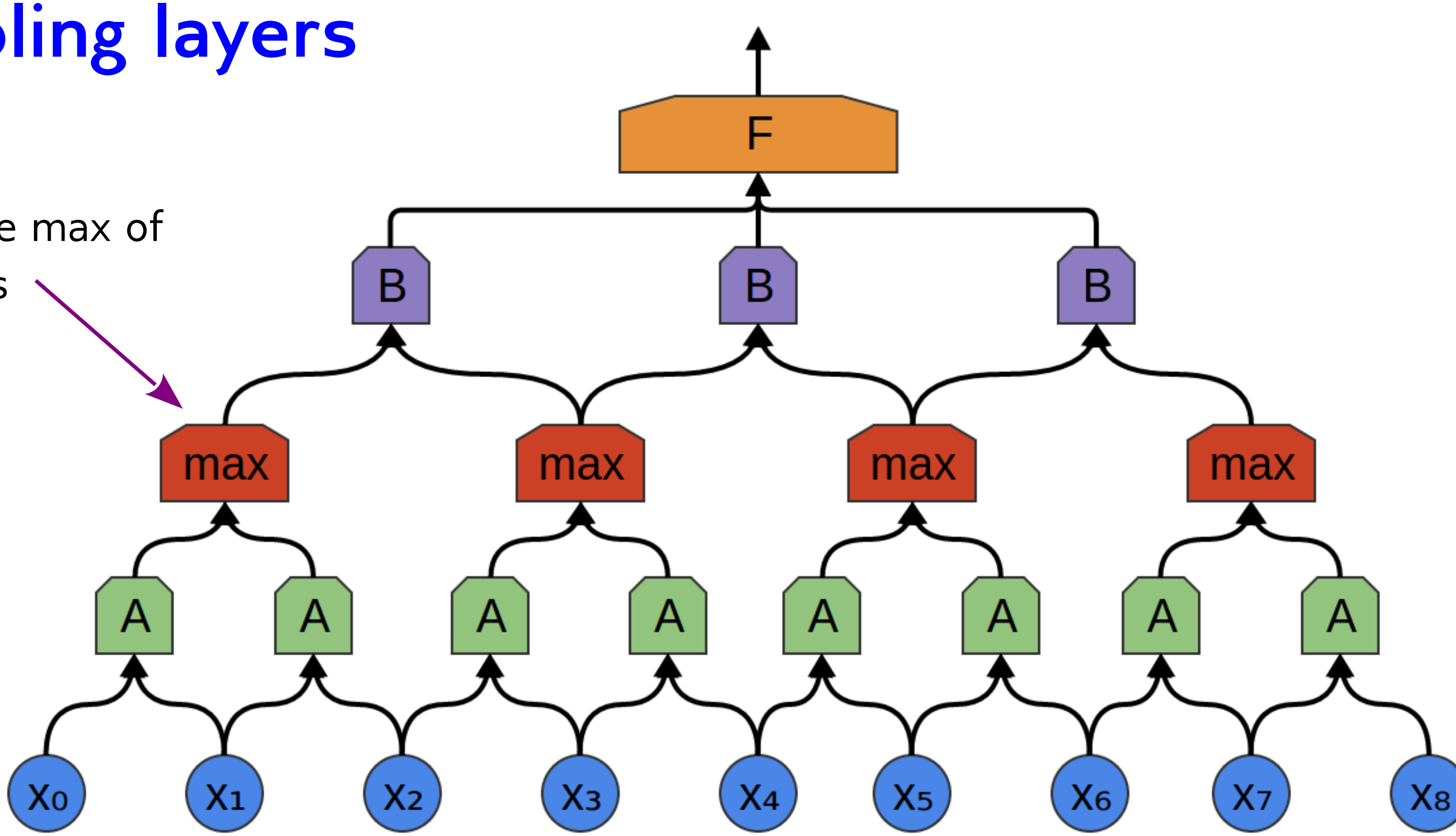


Christopher Olah

What is a Convolutional Neural Network?

- To make the network robust to small translations in detected features, and to reduce the amount of redundant data fed into higher layers, we introduce **pooling layers**

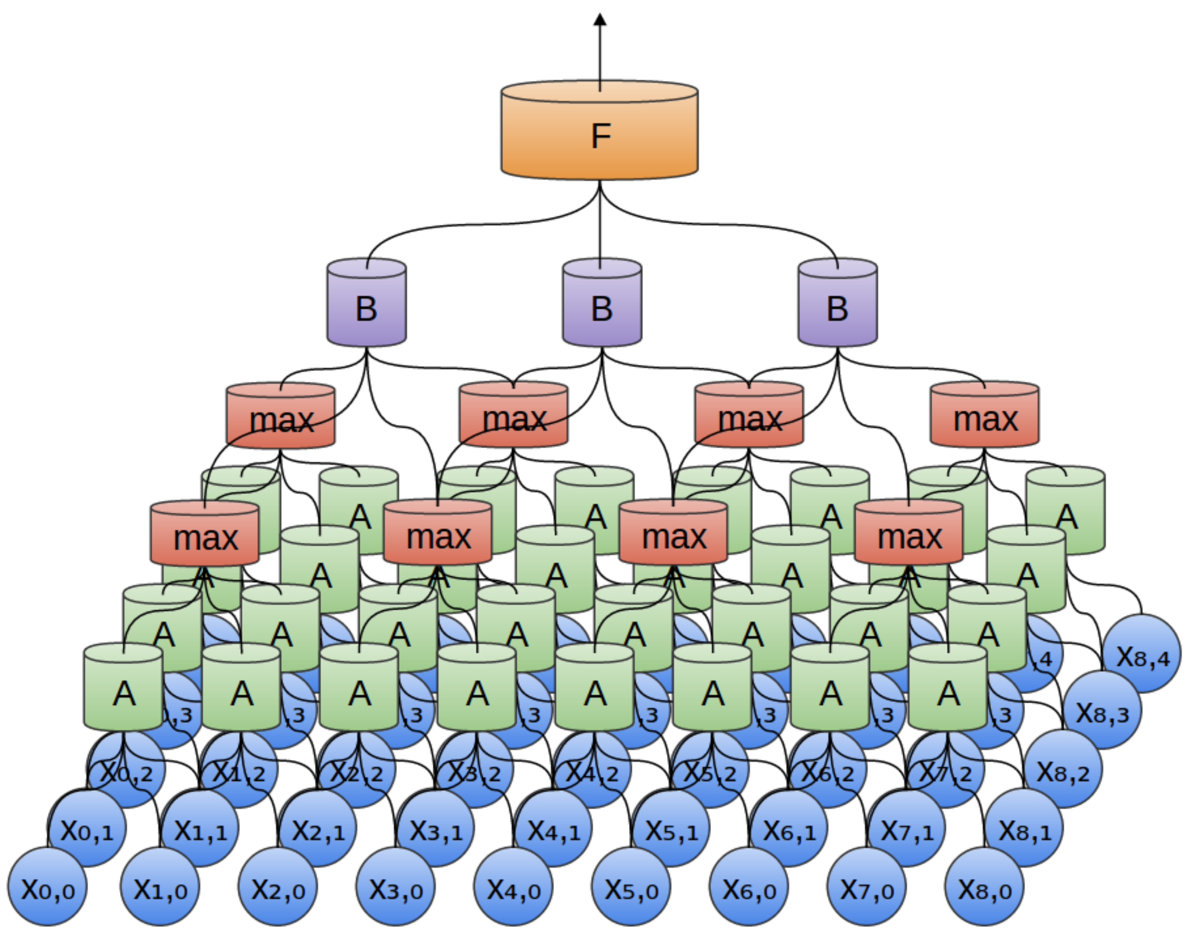
Return the max of the inputs



Christopher Olah

What is a Convolutional Neural Network?

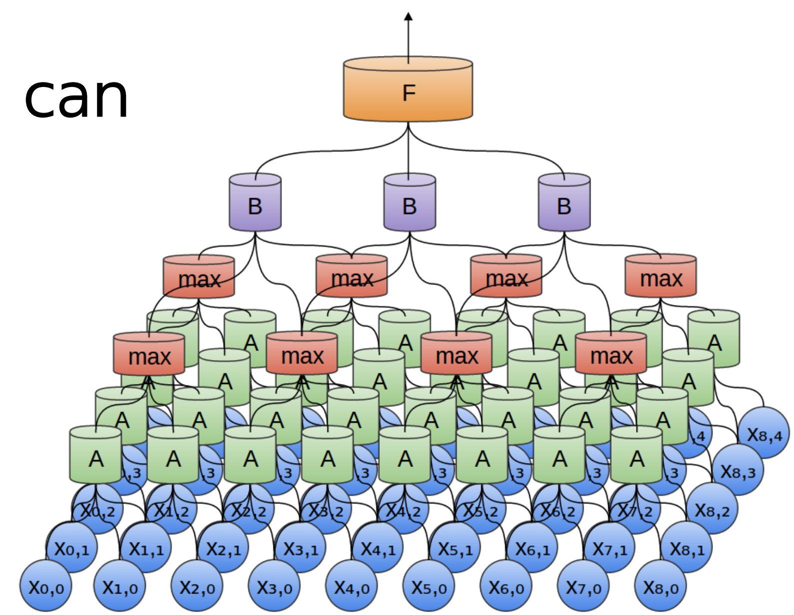
- The signal can be 2D or 3D: the filters are now also 2D/3D, but it's all essentially the same



Christopher Olah

What is a Convolutional Neural Network?

- The function computed by this gigantic model is **differentiable*** w.r.t. the weights
  - Given training data and a **loss function** measuring the deviation between predicted and actual values, we can optimize the weights by gradient descent
  - The gradient of the loss function can be found efficiently by a method called **back-propagation**

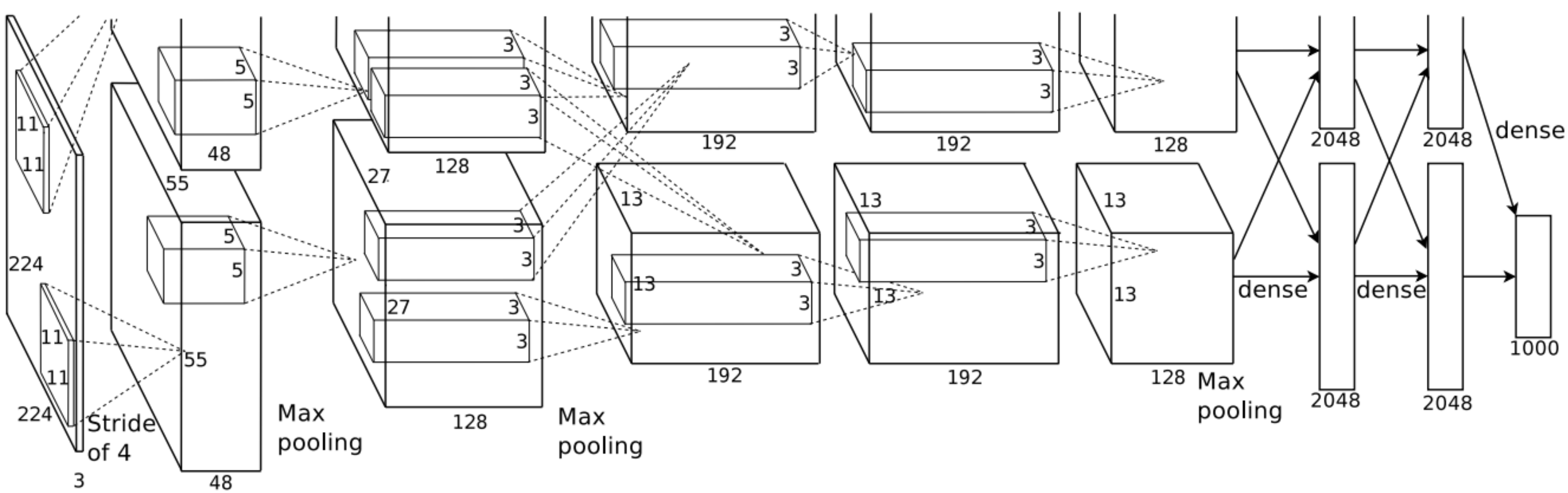


* nearly everywhere

Christopher Olah

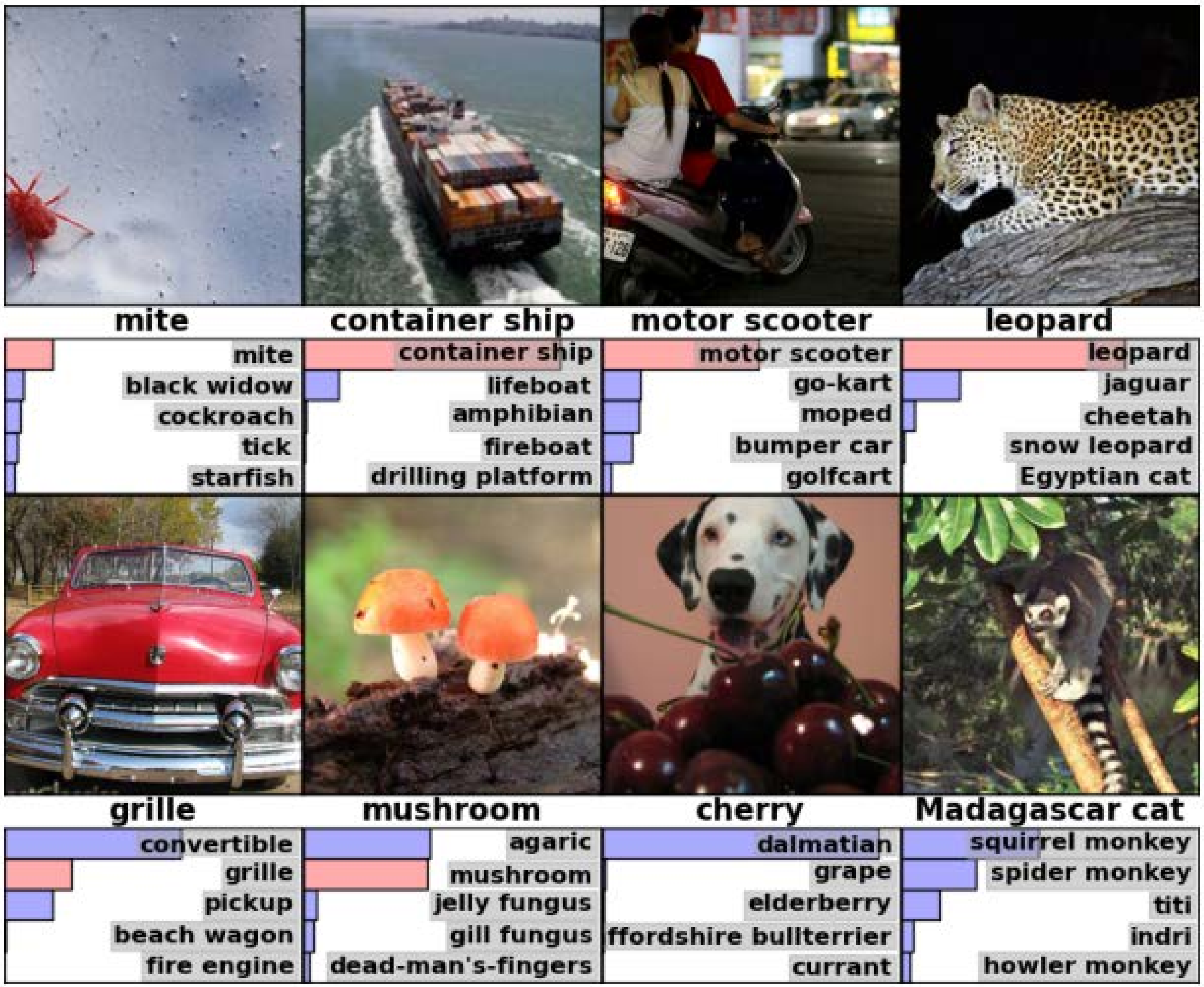
A real-world CNN

- 5 convolutional layers, 3 max-pooling layers, 3 fully-connected layers
- ~60 million parameters (despite the weight sharing!)



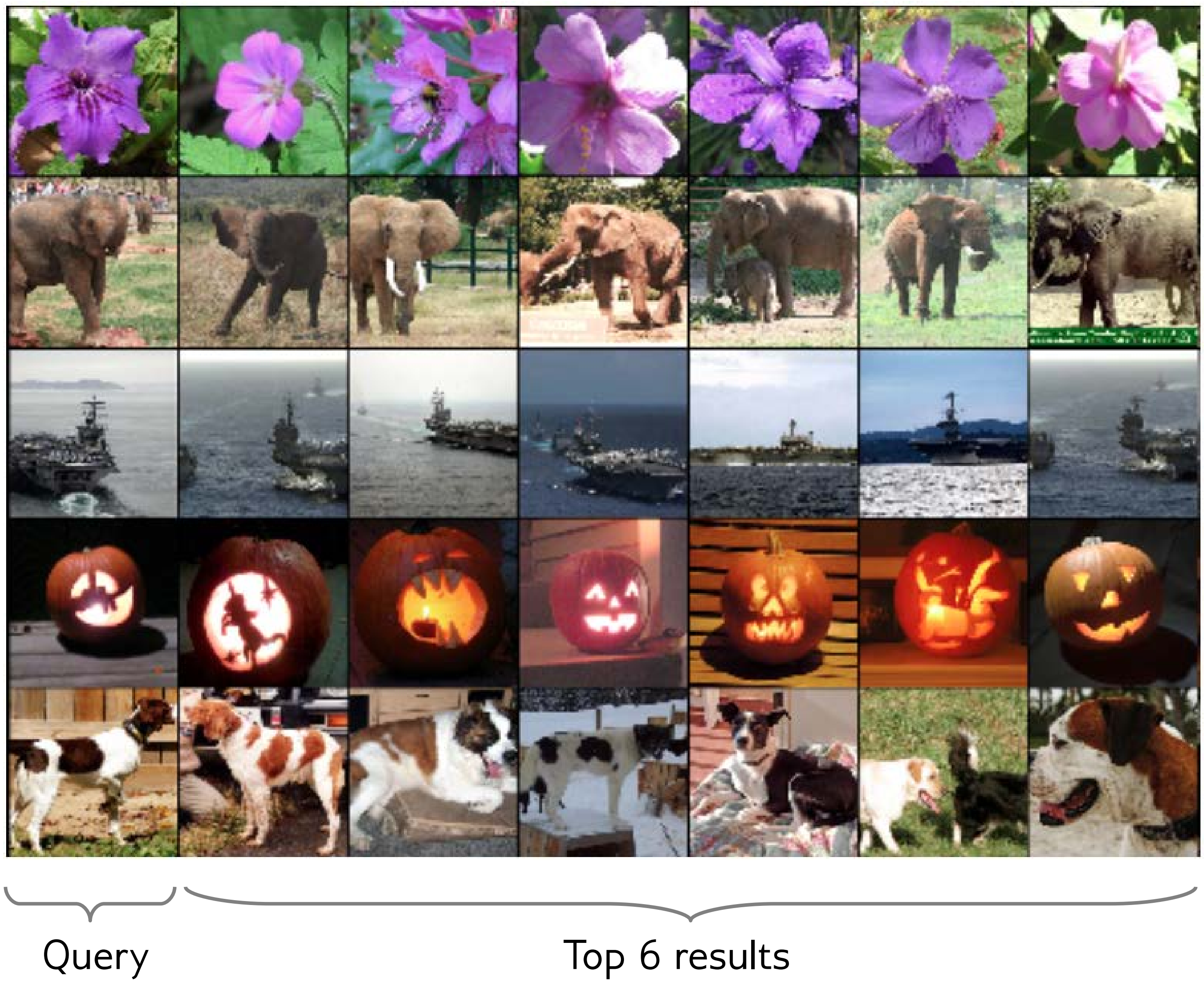
Krizhevsky, Sutskever and Hinton, 2012

Using the CNN for **classification**



Krizhevsky, Sutskever and Hinton, 2012

Using the CNN for **retrieval**



The descriptor is the vector of neuron activations in the second last layer

Krizhevsky, Sutskever and Hinton, 2012

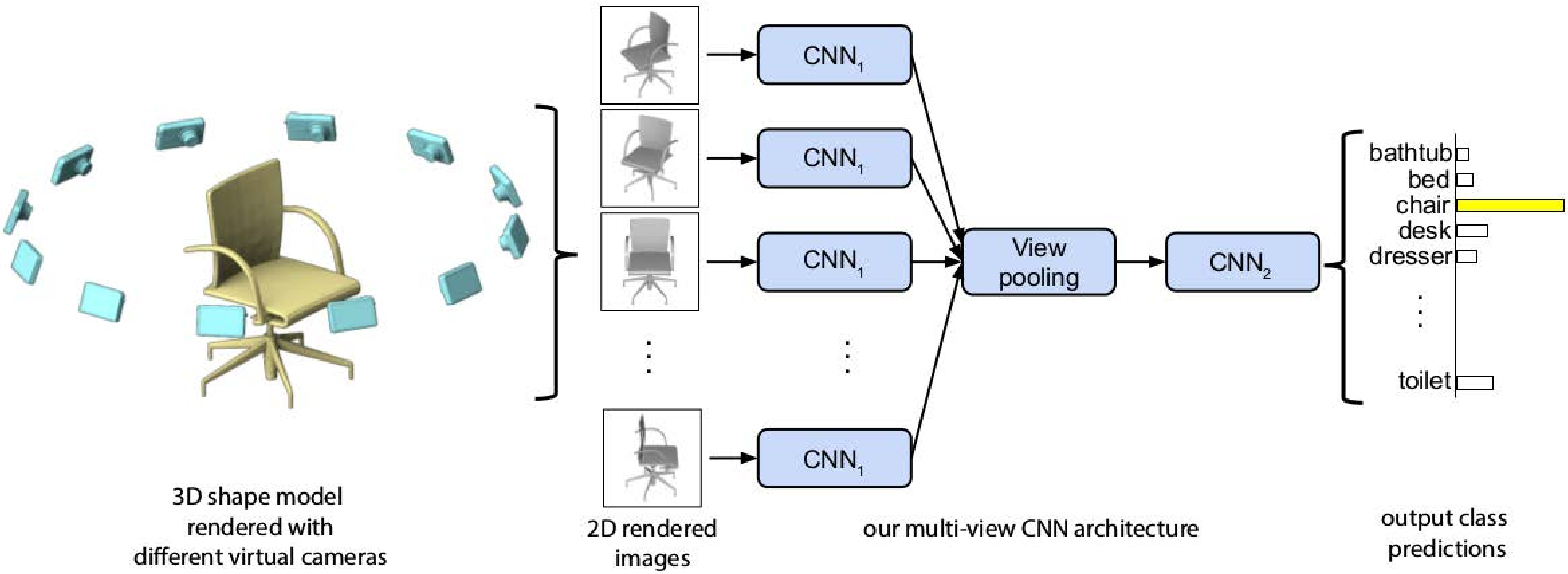
Image CNN for 3D shapes

- Let's take a CNN trained on a (huge) image database, and use it to analyze views of 3D shapes
  - **Render** a 3D shape from an arbitrary viewpoint
  - Pass it through the **pre-trained CNN** and take the neuron activations in the second-last layer as the descriptor
  - For more accuracy, **fine-tune** the network on a training set of rendered shapes before testing
- Just this alone, with a single view (from an unknown direction) of the shape, bumps up the mAP retrieval accuracy (area under PR curve) on a 40-class, 12K-shape collection from 40.9% (LFD) to **61.7%**.
  - An LFD-like approach with 12 views/shape further improves to **62.8%**

Su et al., "Multi-view Convolutional Neural Networks for 3D Shape Recognition", 2015

Combining Views

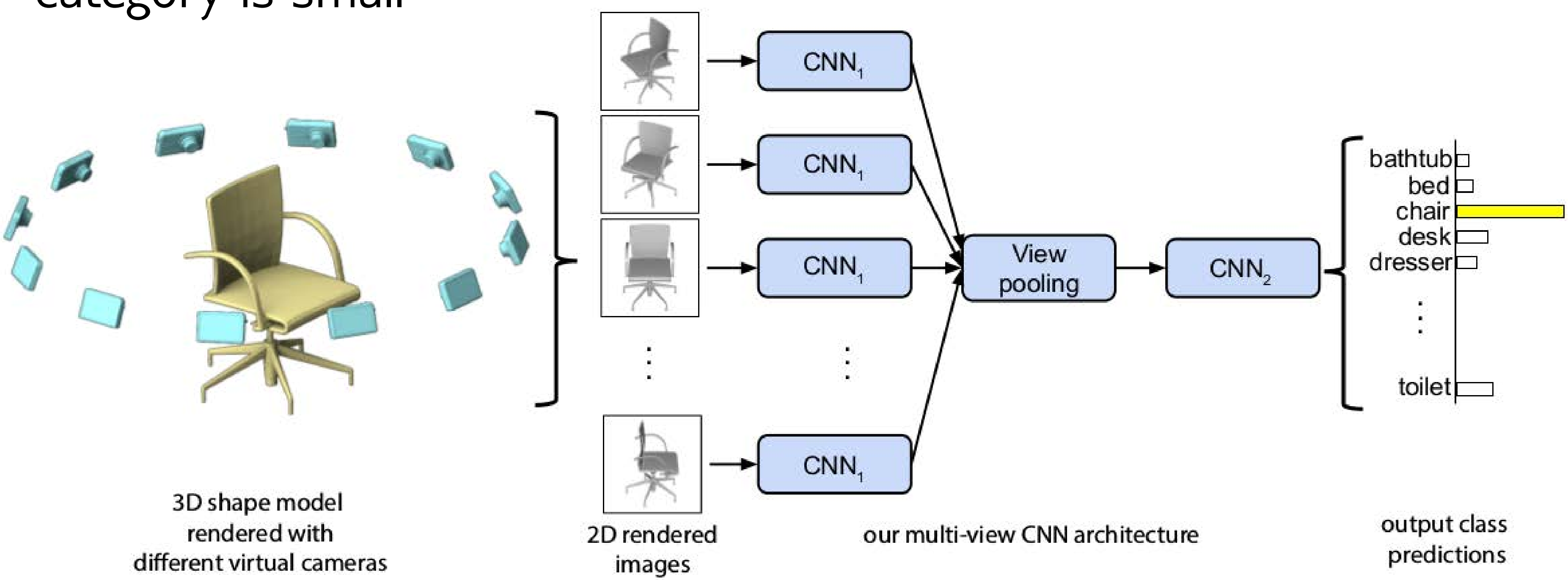
- A smarter way to aggregate information from multiple views
  - Take the output signal of the last convolutional layer of the base network (CNN_1) from each view, and combine them, element-by-element, using a max-pooling operation
  - Pass this **view-pooled** signal through the rest of the network (CNN_2)



Su et al., "Multi-view Convolutional Neural Networks for 3D Shape Recognition", 2015

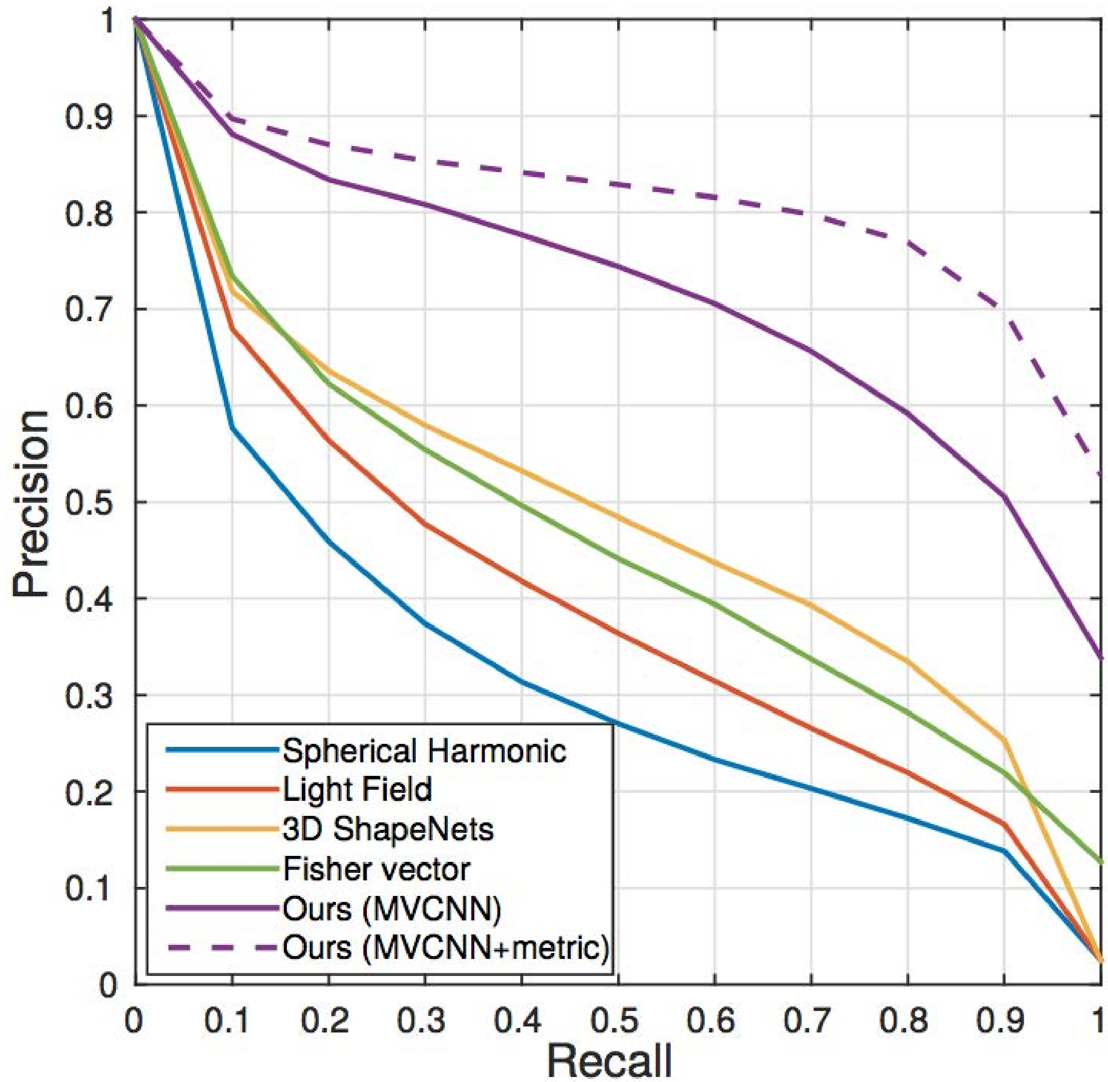
Combining Views

- The view-pooled CNN can still be trained (in exactly the same way) using back-propagation and gradient descent
- For retrieval, the descriptor from the second-last layer can be further tuned by learning a Mahalanobis metric (a projection of the descriptors) where the distance between shapes of the same training category is small



Su et al., "Multi-view Convolutional Neural Networks for 3D Shape Recognition", 2015

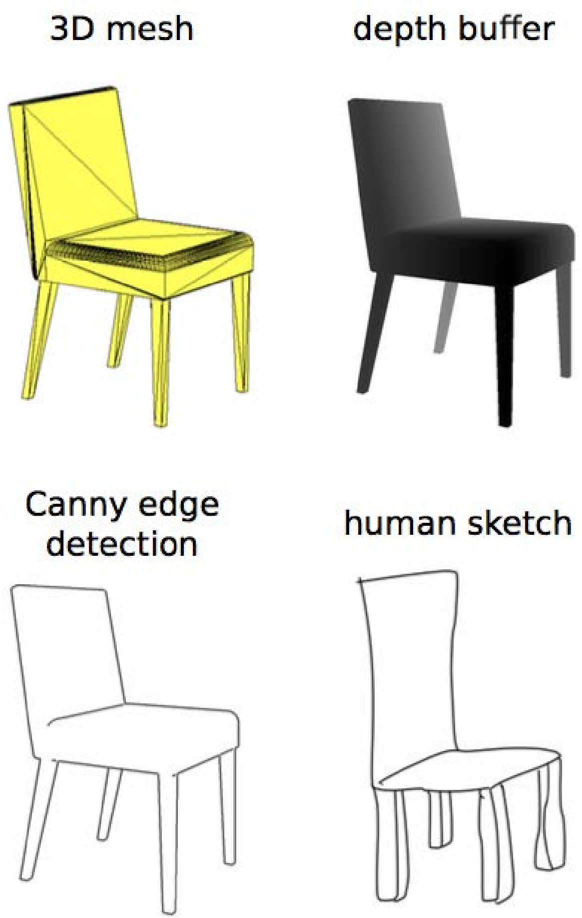
How well does this work?

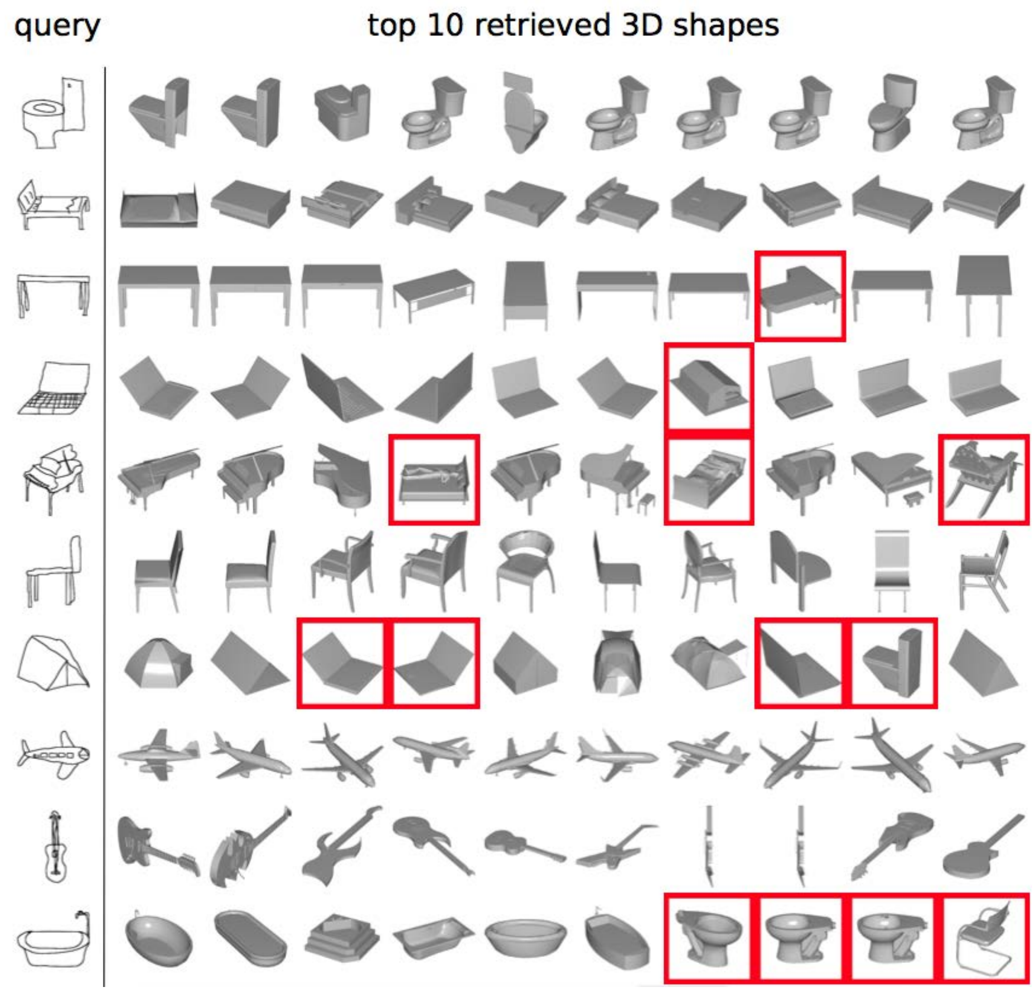


Su et al., “Multi-view Convolutional Neural Networks for 3D Shape Recognition”, 2015

A side benefit of view-based representations

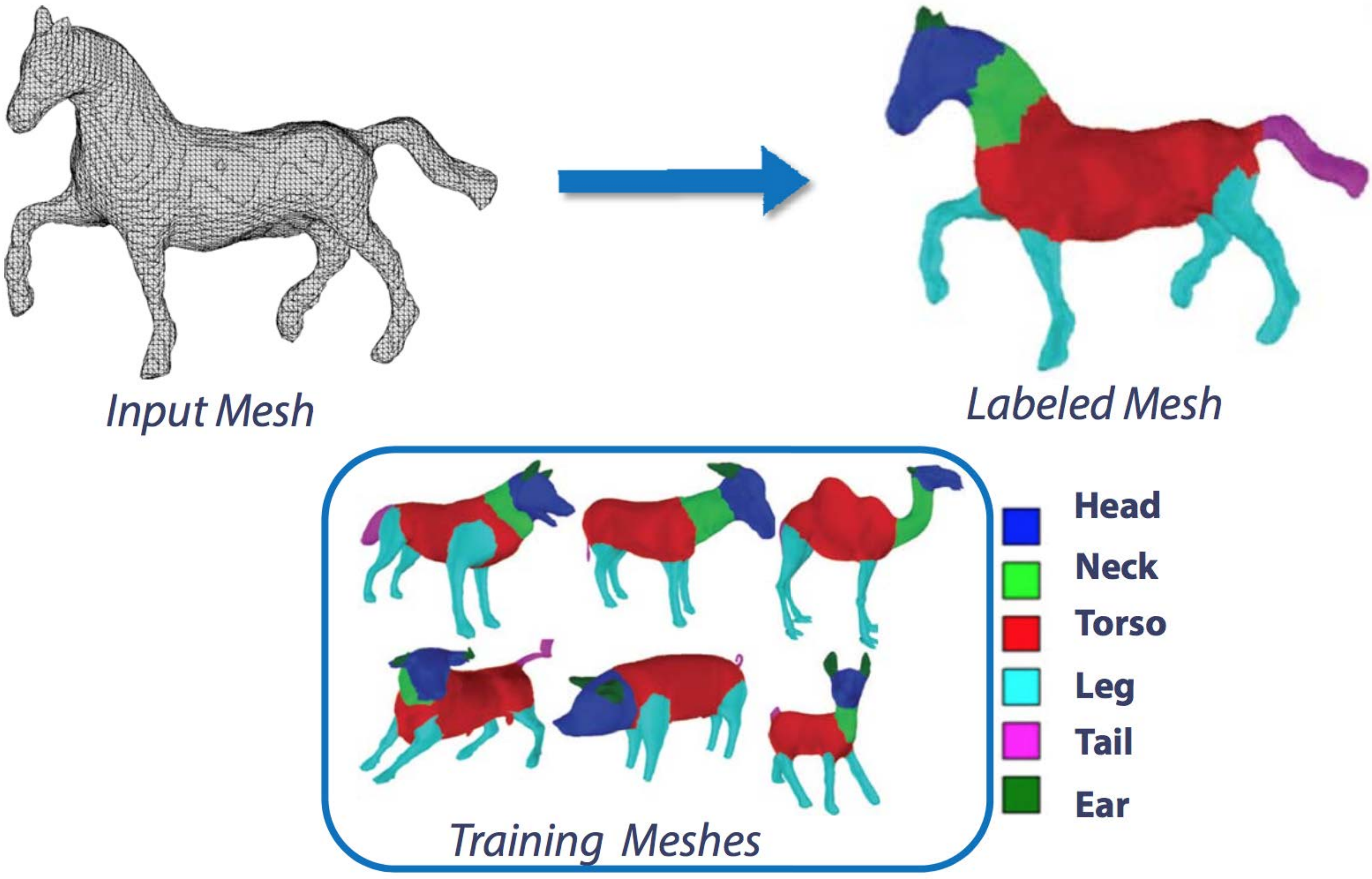
- The MVCNN can be fine-tuned to retrieve 3D models based on hand-drawn 2D sketches





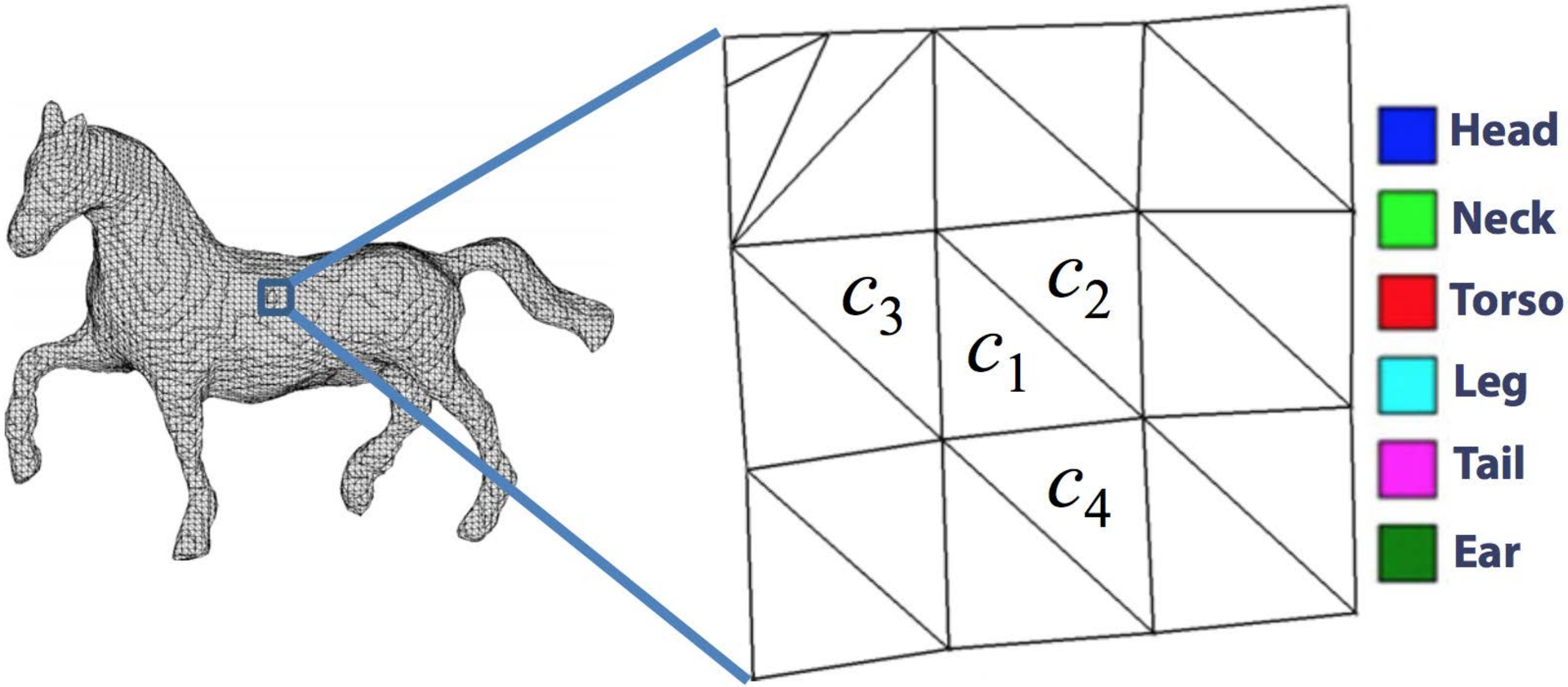
Global descriptors enable retrieval.
Let's look at an application enabled by good ***local*** descriptors.

Shape Segmentation and Labeling



Kalogerakis, Hertzmann and Singh, 2010

Shape Segmentation and Labeling

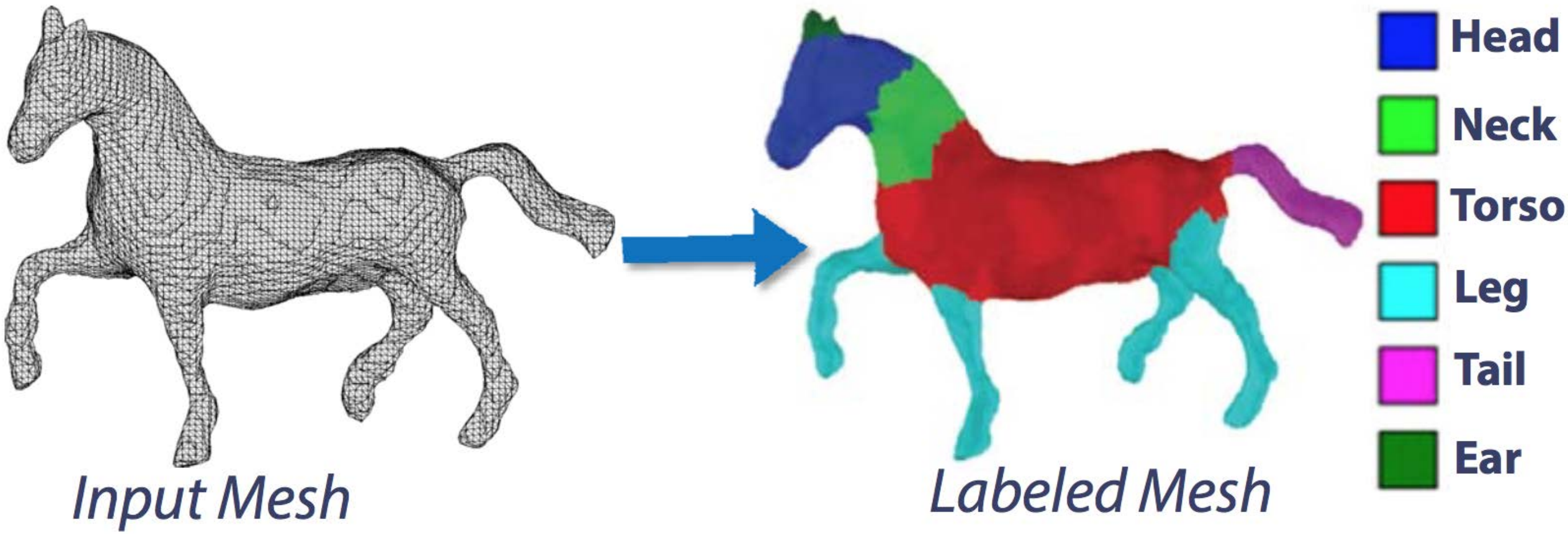


$$c_1, c_2, c_3 \in C$$

$$C = \{\, \textit{head}, \textit{neck}, \textit{torso}, \textit{leg}, \textit{tail}, \textit{ear} \,\}$$

Kalogerakis, Hertzmann and Singh, 2010

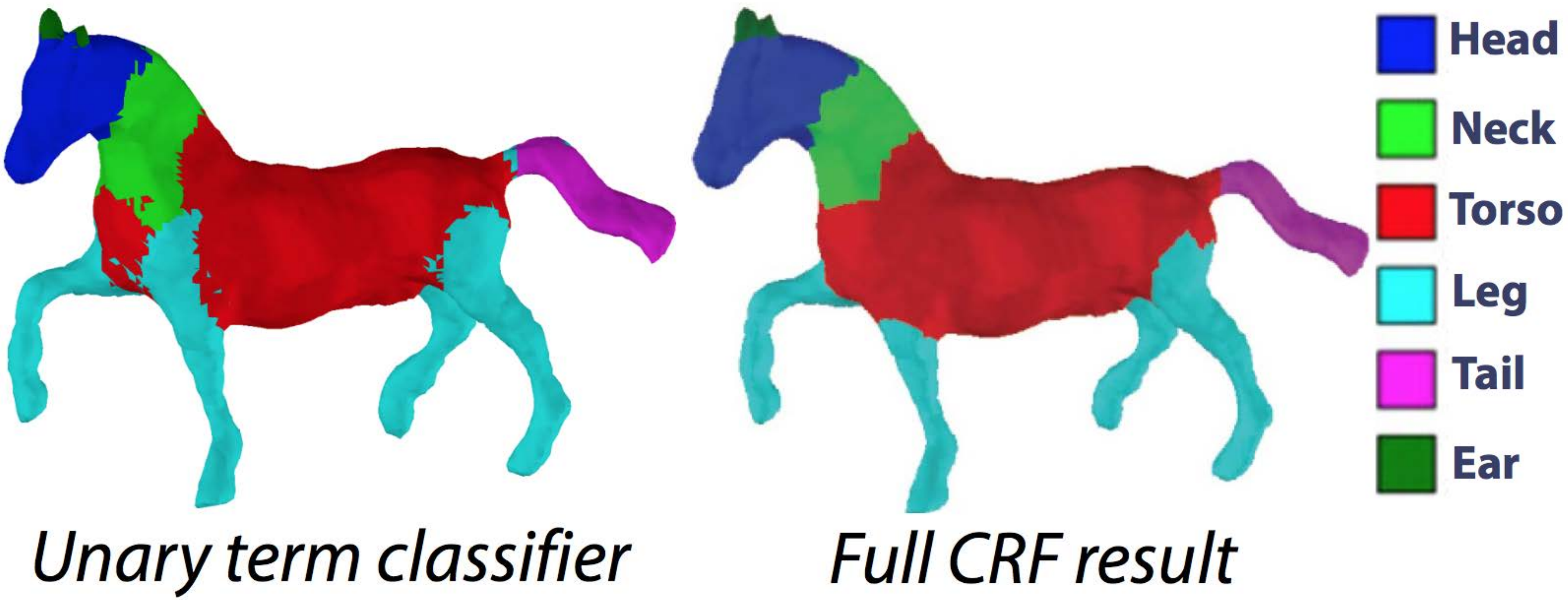
Conditional Random Field for Segmentation and Labeling



$$c^* = \arg\min_{\mathbf{c}} \left\{ \sum_{i} \underbrace{\alpha_i \mathrm{E}_1(c_i;\mathbf{x_i})}_{\text{Unary term}} + \sum_{i,j} l_{ij} \underbrace{\mathrm{E}_2(c_i, c_j; \mathbf{y_{ij}})}_{\text{Pairwise term}} \right\}$$

Kalogerakis, Hertzmann and Singh, 2010

Effect of the pairwise term



Kalogerakis, Hertzmann and Singh, 2010

View-based local descriptors?

- CNNs can also yield local descriptors
- If multi-view CNNs dramatically improve retrieval accuracy, can they also improve segmentation accuracy?
- The answer appears to be yes (more details coming soon!)

"High-Level" Geometric Analysis

- What **type** of object is this?
- How can we **generate** more objects like this?
- What **attributes** does it have?
- What **functions** does it serve?

Ake Axelsson

Outline

- Learning shape structure
  - **Probabilistic models** of shape

Outline

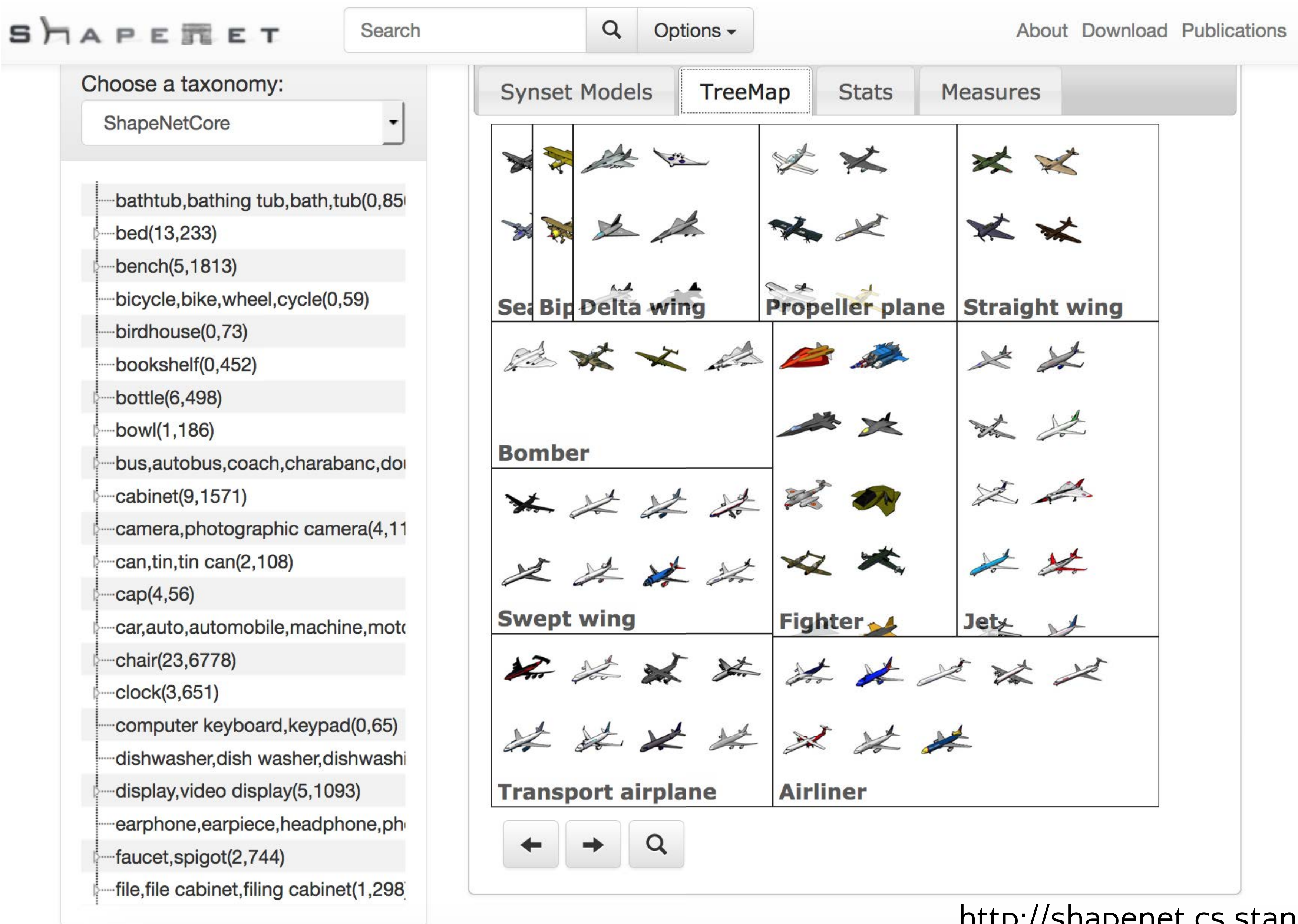
- Learning shape structure
  - **Probabilistic models** of shape

- Learning shape semantics
  - Semantic **attributes** (*scary, artistic,* ...)
  - Mechanical **function** (*this airplane should fly*...)
  - Human **interaction** (*sit comfortably in a chair*...)

What is the role of data?

Google/Trimble 3D Warehouse (~millions of downloadable models)

What is the role of data?



http://shapenet.cs.stanford.edu

What is the role of data?

- **Reuse** (of existing components)
- **Training** (of computational models)
- **Inspiration** (for new designs)

Outline

- Learning shape structure
  - **Probabilistic models** of shape

- Learning shape semantics
  - Semantic **attributes** (*scary, artistic,* ...)
  - Mechanical **function** (*this airplane should fly*...)
  - Human **interaction** (*sit comfortably in a chair*...)

Shape spaces should be...

- **General**
  - Topological/geometric/configurational variety
- **Probabilistic**
  - Some shapes are more plausible than others
- **Generative**
  - Can be used to produce new shapes
- **Meaningfully Parametrized**
  - Design intent readily maps to "suitable" shapes

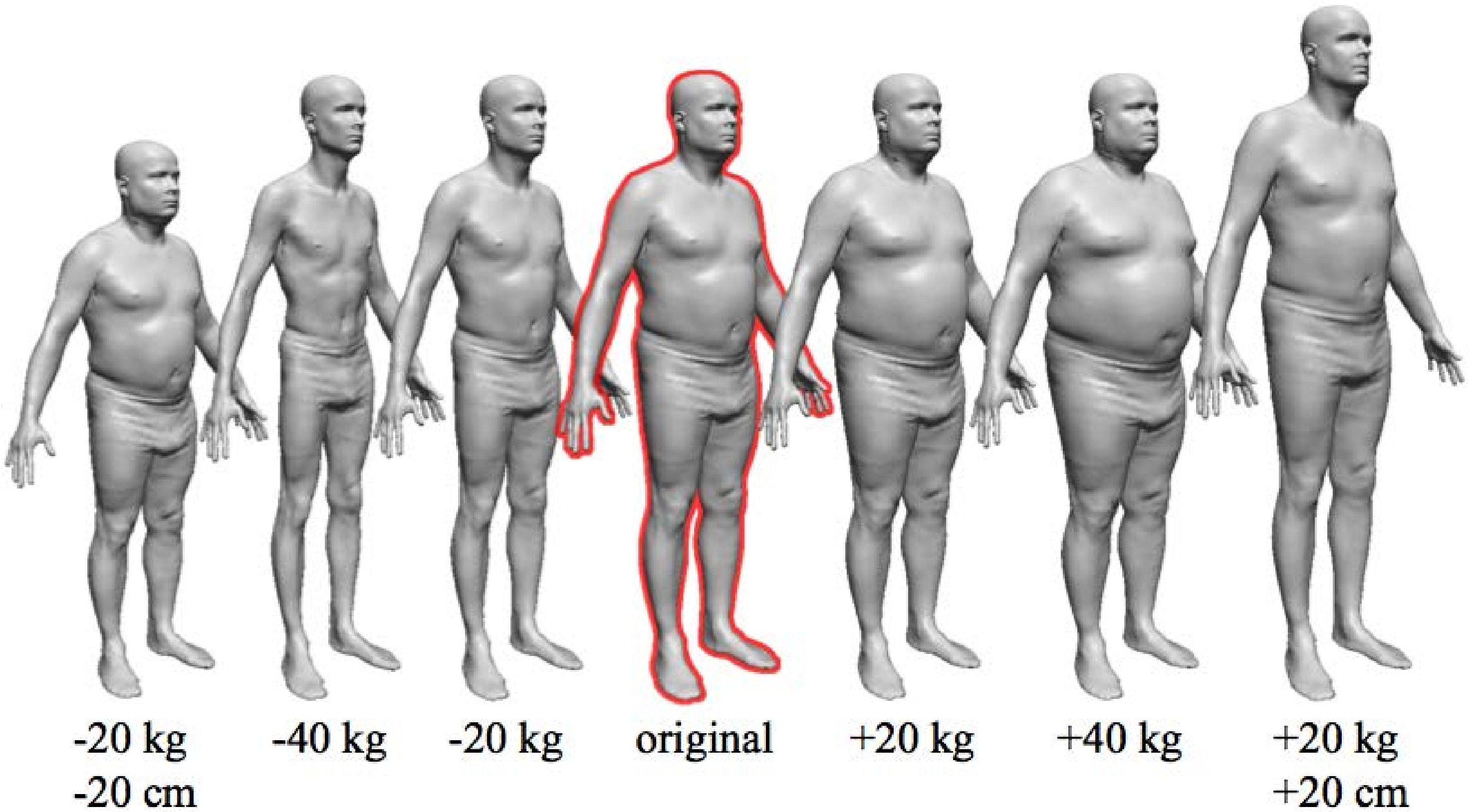
Shape Space: Maya

Sequences of commands to Maya/AutoCAD/ZBrush...

Generality:	**High**	Meaningful parametrization:	**No**
Probabilistic:	**No**	Data-driven:	**No**

Shape Space: Deformable Template

(one topology, plus parameters for body type)



Generality: **Low**

Probabilistic: **Yes**

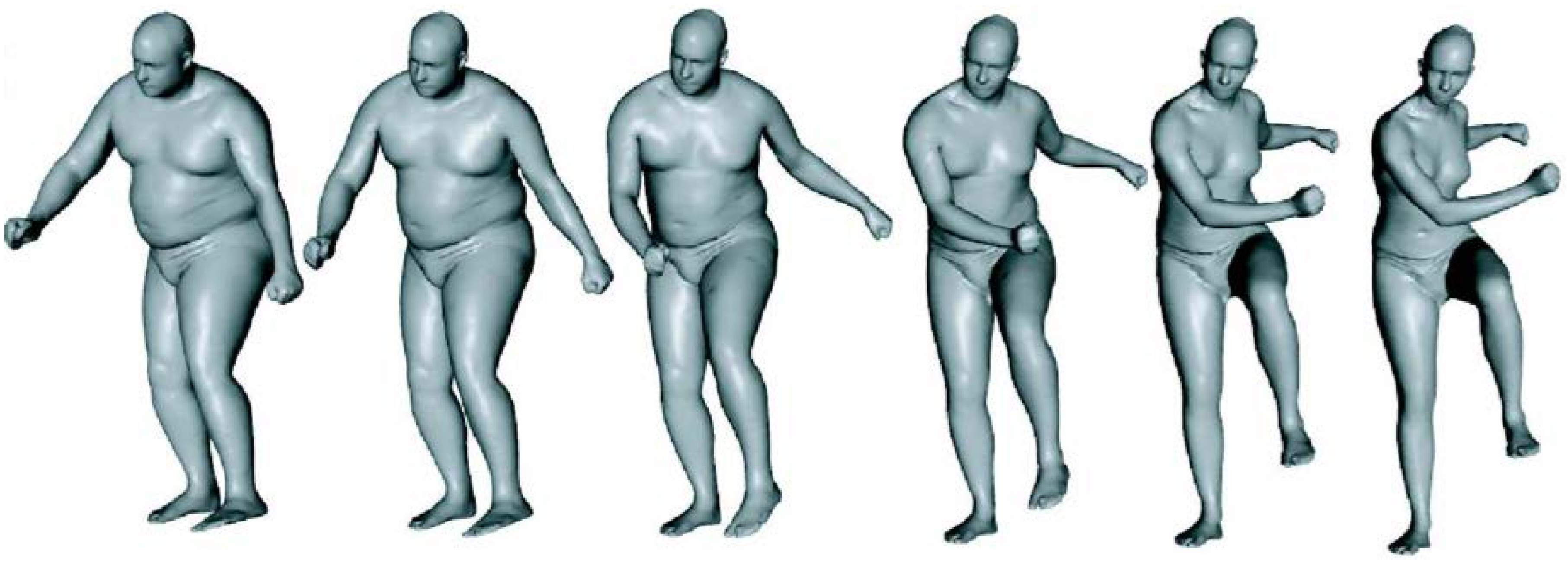
Meaningful parametrization: **Moderate**

Data-driven: **Yes**

Allen, Curless and Popovic, 2003

Shape Space: Deformable Template

(one topology, plus parameters for both body type and pose)



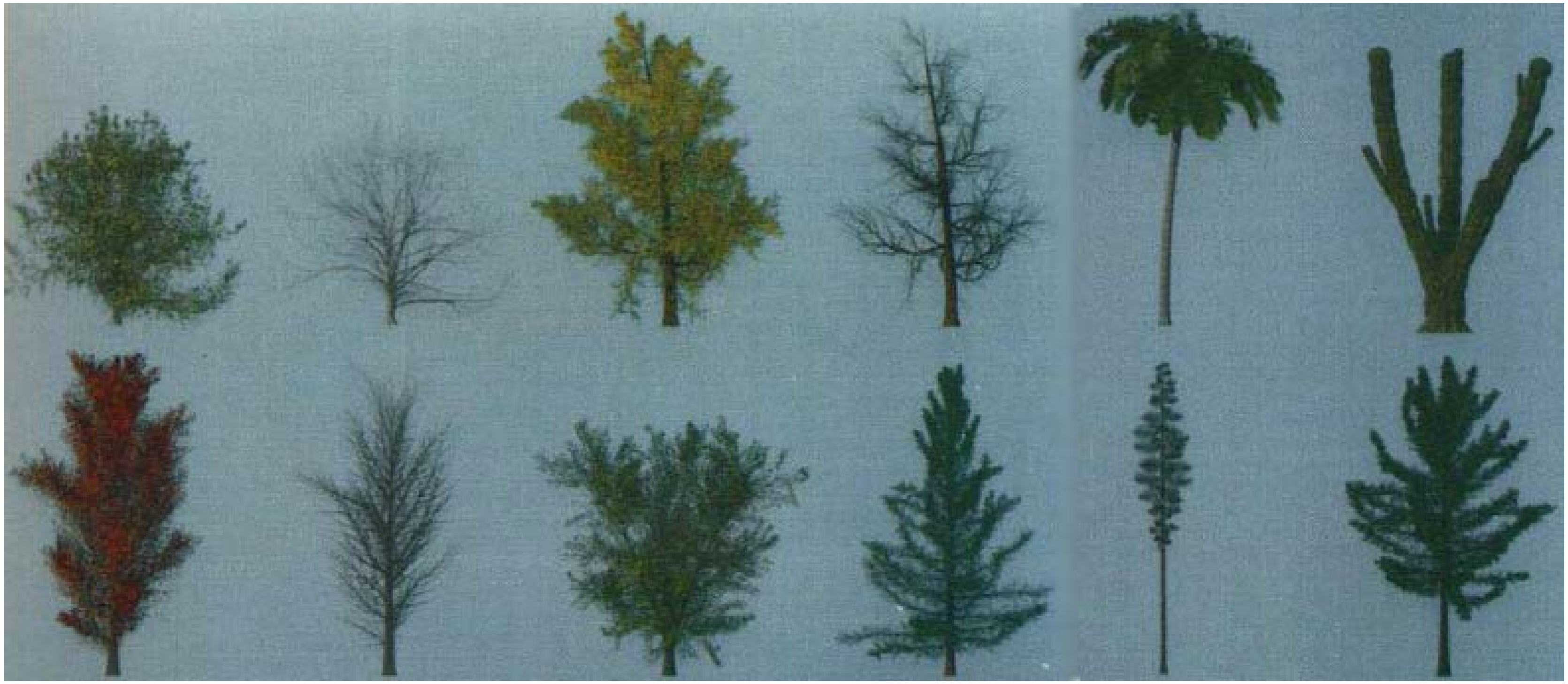
Generality: Low-ish
Probabilistic: Yes

Meaningful parametrization: Moderate
Data-driven: Yes

Anguelov et al., 2005

Shape Space: Parametrized Procedure

(fixed set of parameters)



Generality: **Moderate**

Probabilistic: **No**

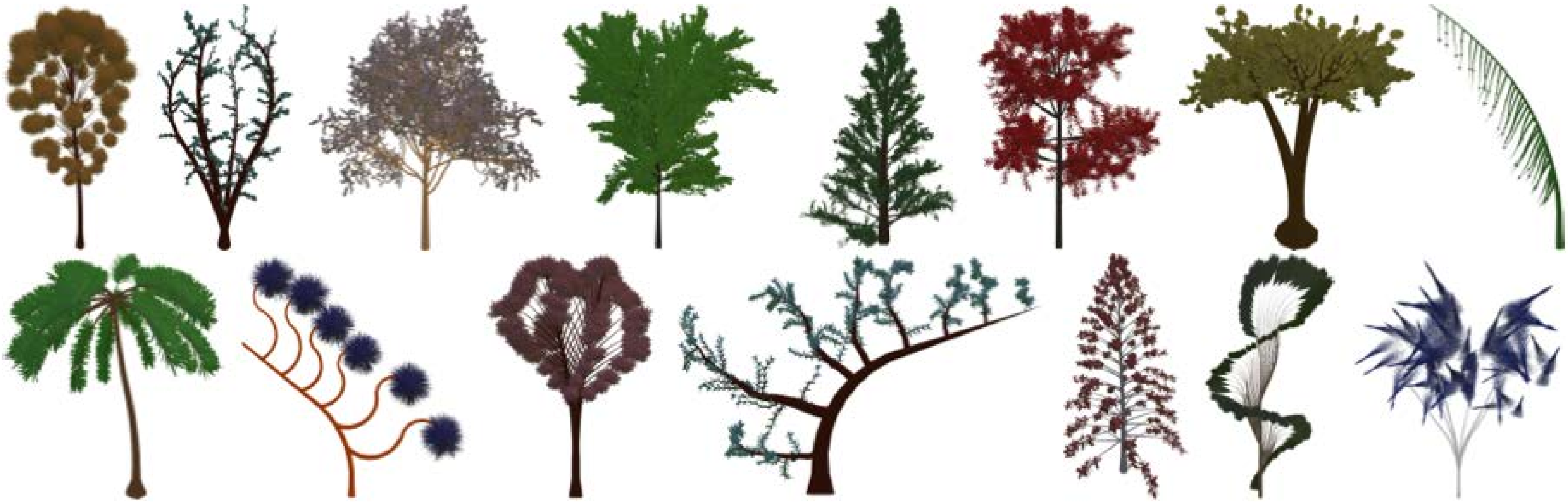
Meaningful parametrization: **Yes**

Data-driven: **No**

Weber and Penn, 1995

Shape Space: Probabilistic Procedure

(probability distribution on parameters)



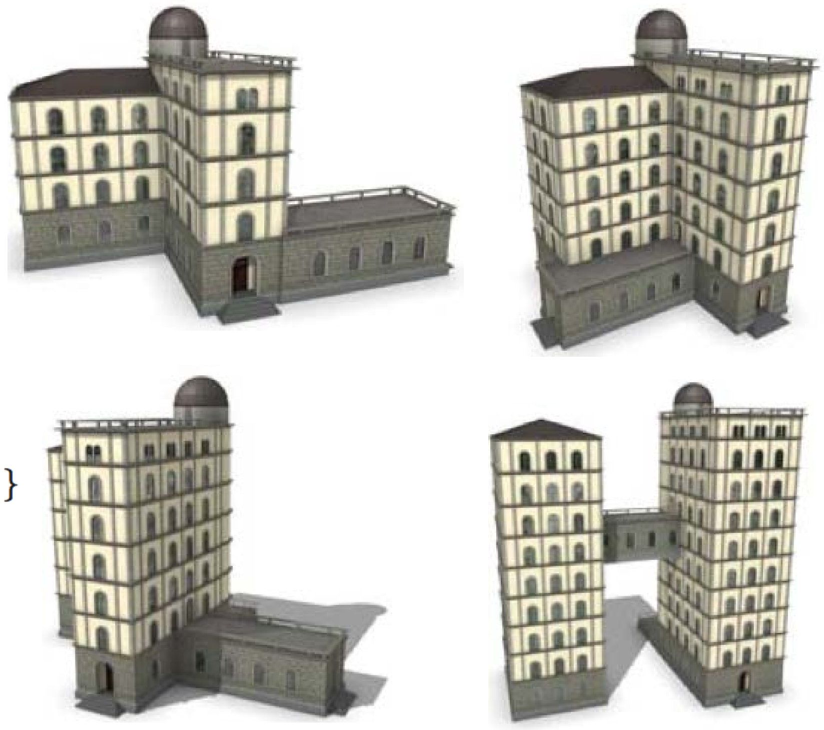
Generality:	**Moderate**	Meaningful parametrization:	**Yes**
Probabilistic:	**Yes**	Data-driven:	**Partially**

Talton et al., 2009

Shape Space: Probabilistic Grammar

(hierarchical generation)

```
PRIORITY 1:
1:  footprint ⇝ S(1r,building_height,1r) facades
      T(0,building_height,0) Roof("hipped",roof_angle){ roof }
PRIORITY 2:
2:  facades ⇝ Comp("sidefaces"){ facade }
3:  facade : Shape.visible("street")
      ⇝ Subdiv("X",1r,door_width*1.5){ tiles | entrance } : 0.5
      ⇝ Subdiv("X",door_width*1.5,1r){ entrance | tiles } : 0.5
4:  facade ⇝ tiles
5:  tiles ⇝ Repeat("X",window_spacing){ tile }
6:  tile ⇝ Subdiv("X",1r,window_width,1r){ wall |
      Subdiv("Y",2r,window_height,1r){ wall | window | wall } | wall }
7:  window : Scope.occ("noparent") != "none" ⇝ wall
8:  window ⇝ S(1r,1r,window_depth) I("win.obj")
9:  entrance ⇝ Subdiv("X",1r,door_width,1r){ wall |
      Subdiv("Y",door_height,1r){ door | wall } | wall }
10: door ⇝ S(1r,1r,door_depth) I("door.obj")
11: wall ⇝ I("wall.obj")
```



Generality: **Moderate** | Meaningful parametrization: **Yes**

Probabilistic: **Yes** | Data-driven: **Reuse**

Müller et al., 2006

Shape Space: Shape Grammar

(learned from a single example)

Generality: **Moderate**

Probabilistic: **No**

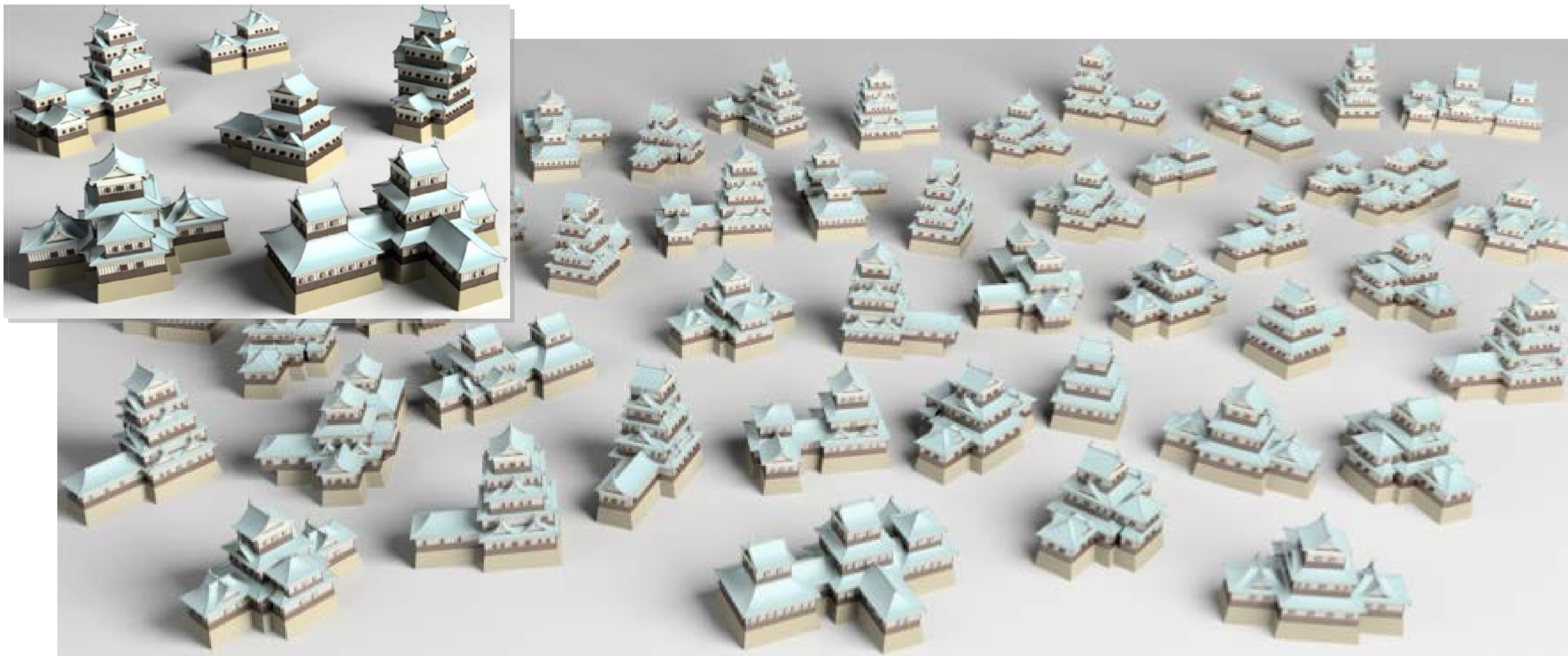
Meaningful parametrization: **Moderate**

Data-driven: **Moderate**

Bokeloh et al. 2010

Shape Space: Probabilistic Grammar

(learned from examples)



Generality:	**Moderate**	Meaningful parametrization:	**Moderate**
Probabilistic:	**Yes**	Data-driven:	**Yes**

Talton et al., 2012

Shape Space: Assembly-Based Modeling

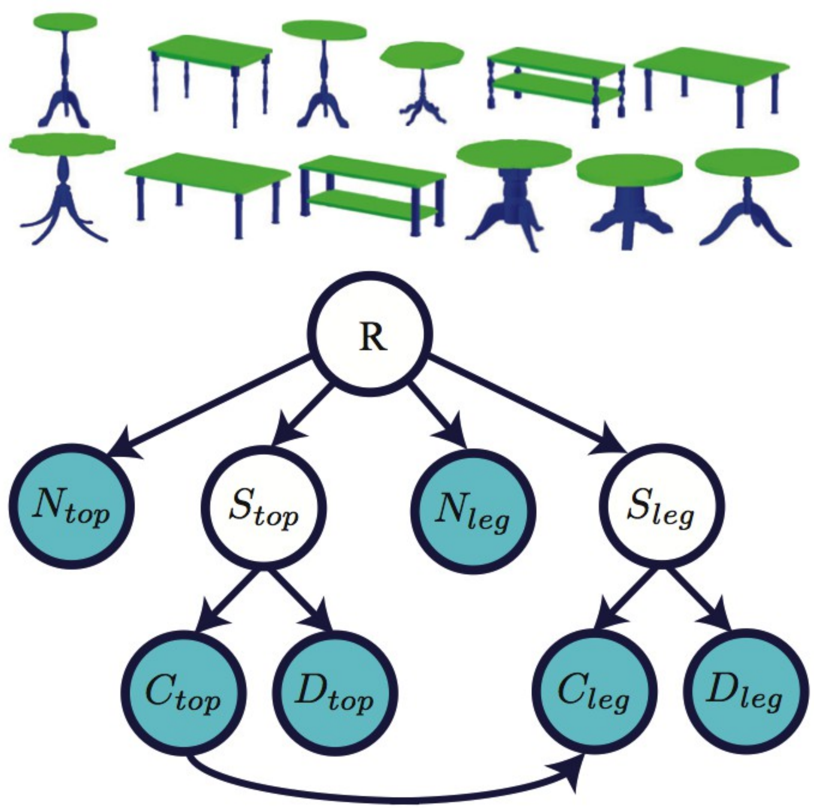
(piece together existing components)

Generality:	**Moderate**	Meaningful parametrization:	**Yes**
Probabilistic:	**No**	Data-driven:	**Reuse**

Spore, Maxis 2008

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

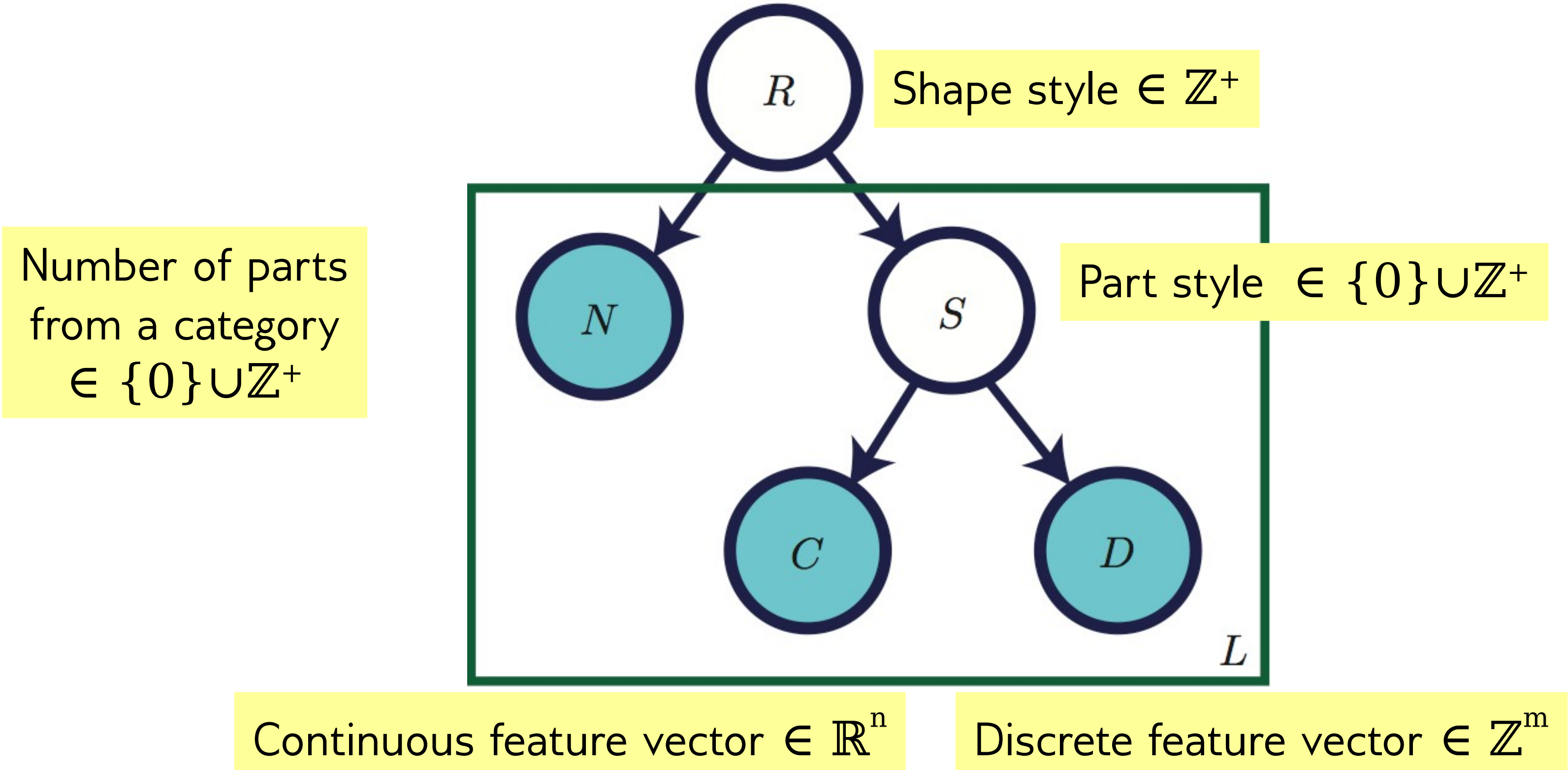


Generality: **Moderate**
Probabilistic: **Yes**
Meaningful parametrization: **Yes**
Data-driven: **Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)



$$P(\mathbf{X}) = P(R) \prod_{l \in \mathcal{L}} \left[P(S_l \mid R) P(N_l \mid R, \pi(N_l)) P(\mathbf{C}_l \mid S_l, \pi(\mathbf{C}_l)) P(\mathbf{D}_l \mid S_l, \pi(\mathbf{D}_l)) \right]$$

Generality:	Moderate	Meaningful parametrization:	Yes
Probabilistic:	Yes	Data-driven:	Yes

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

Learned shape styles

Learned component styles

Generality: **Moderate** Meaningful parametrization: **Yes**
Probabilistic: **Yes** Data-driven: **Yes**

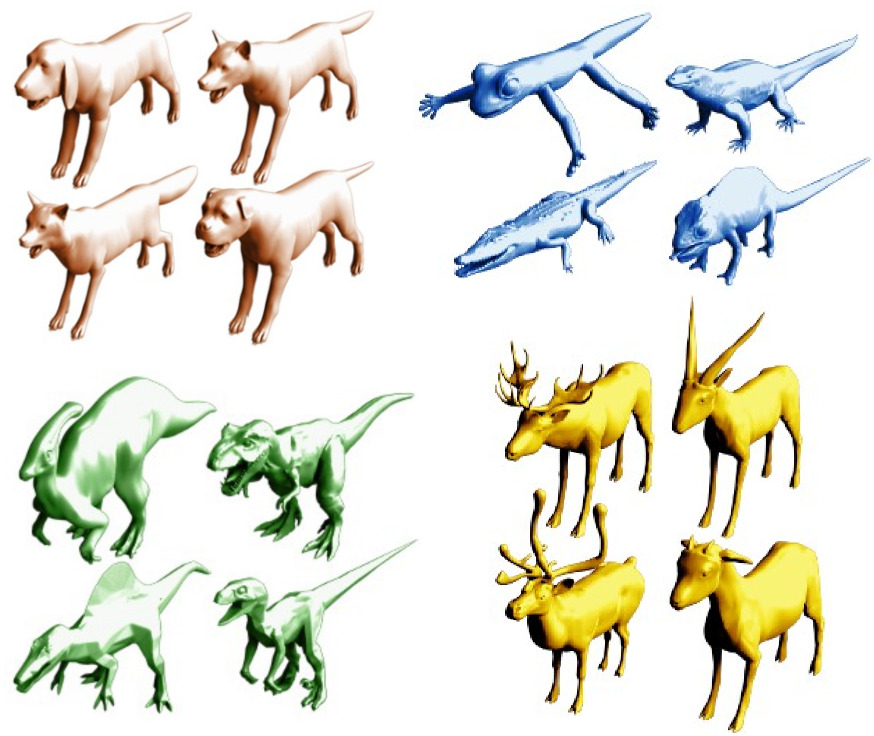
Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

Learned shape styles

Learned component styles



More learned shape “styles”

Generality: **Moderate**

Probabilistic: **Yes**

Meaningful parametrization: **Yes**

Data-driven: **Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

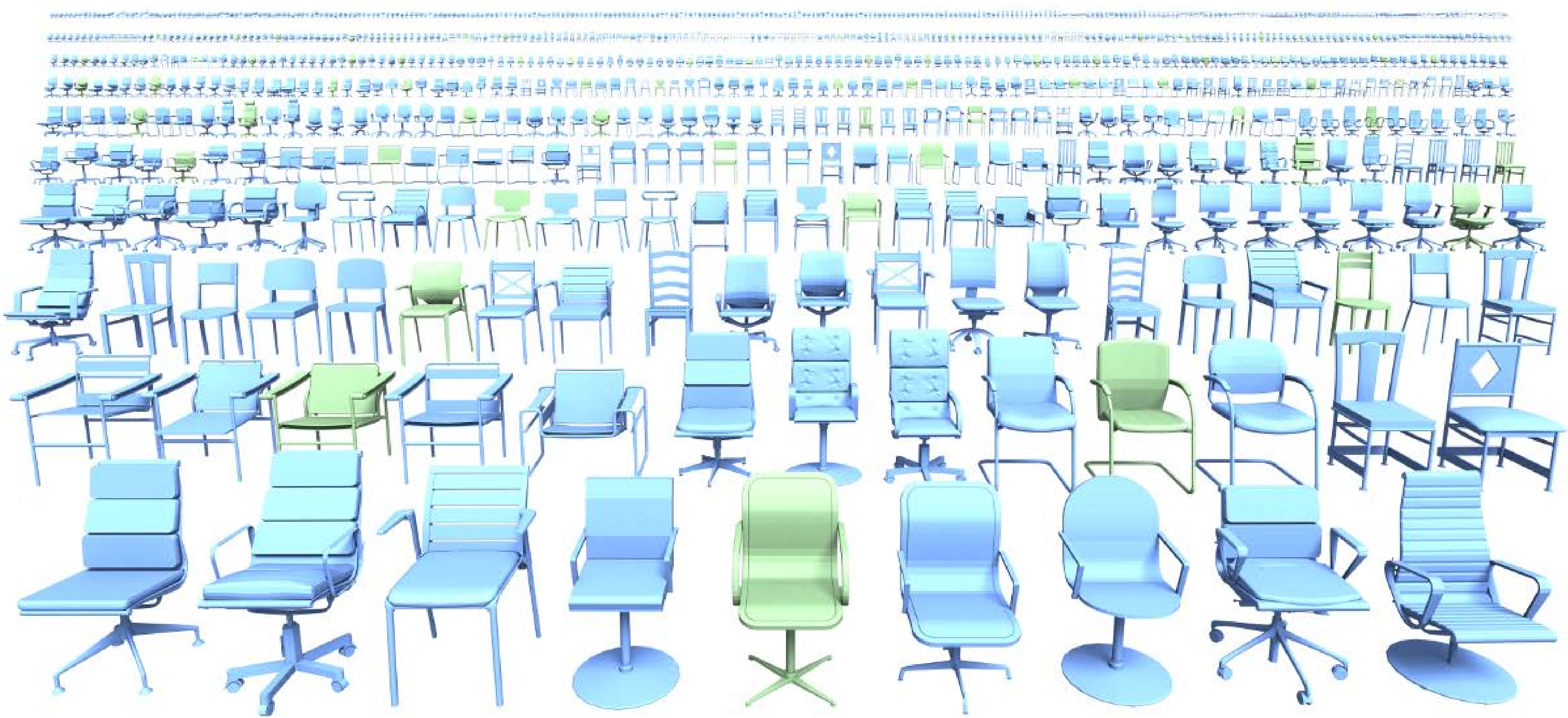
(some assemblies are better than others)

Generality:	**Moderate**	Meaningful parametrization:	**Yes**
Probabilistic:	**Yes**	Data-driven:	**Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

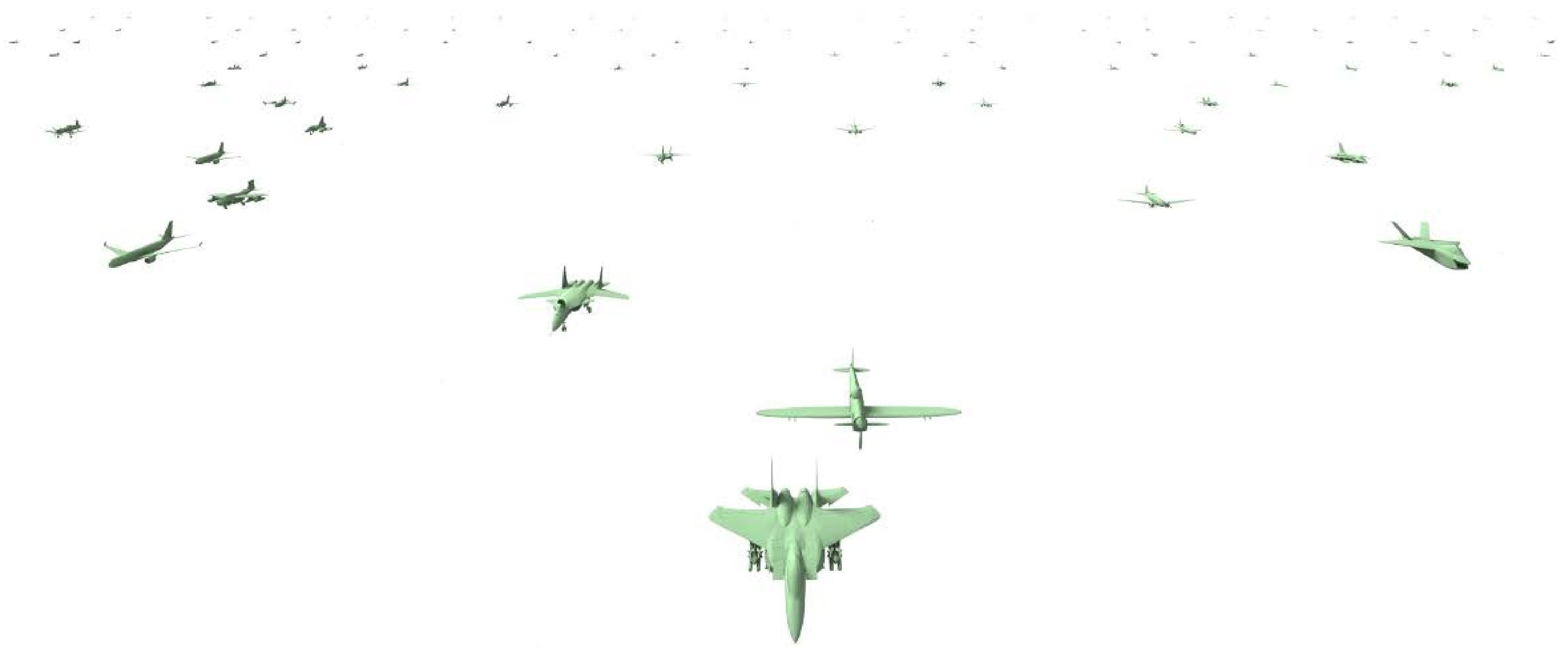


Generality:	Moderate	Meaningful parametrization:	Yes
Probabilistic:	Yes	Data-driven:	Yes

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

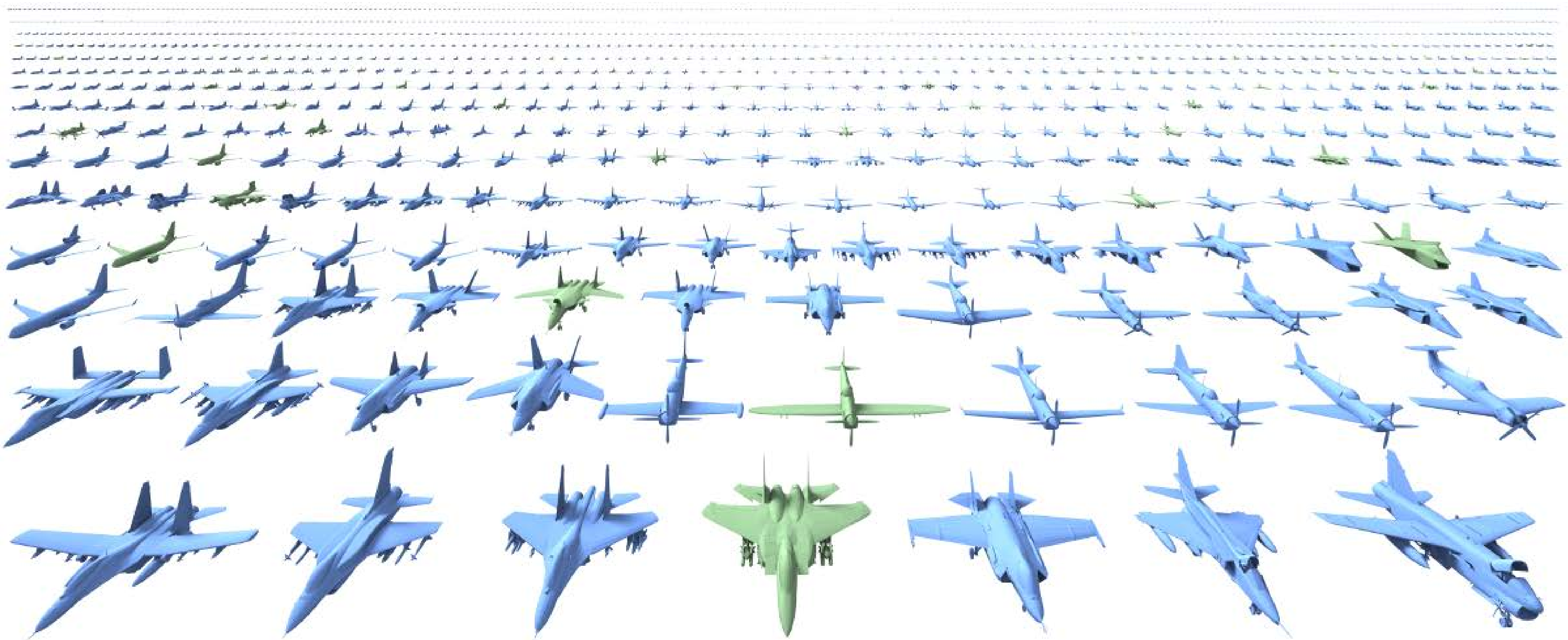


Generality:	Moderate	Meaningful parametrization:	Yes
Probabilistic:	Yes	Data-driven:	Yes

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)



Generality: **Moderate**

Probabilistic: **Yes**

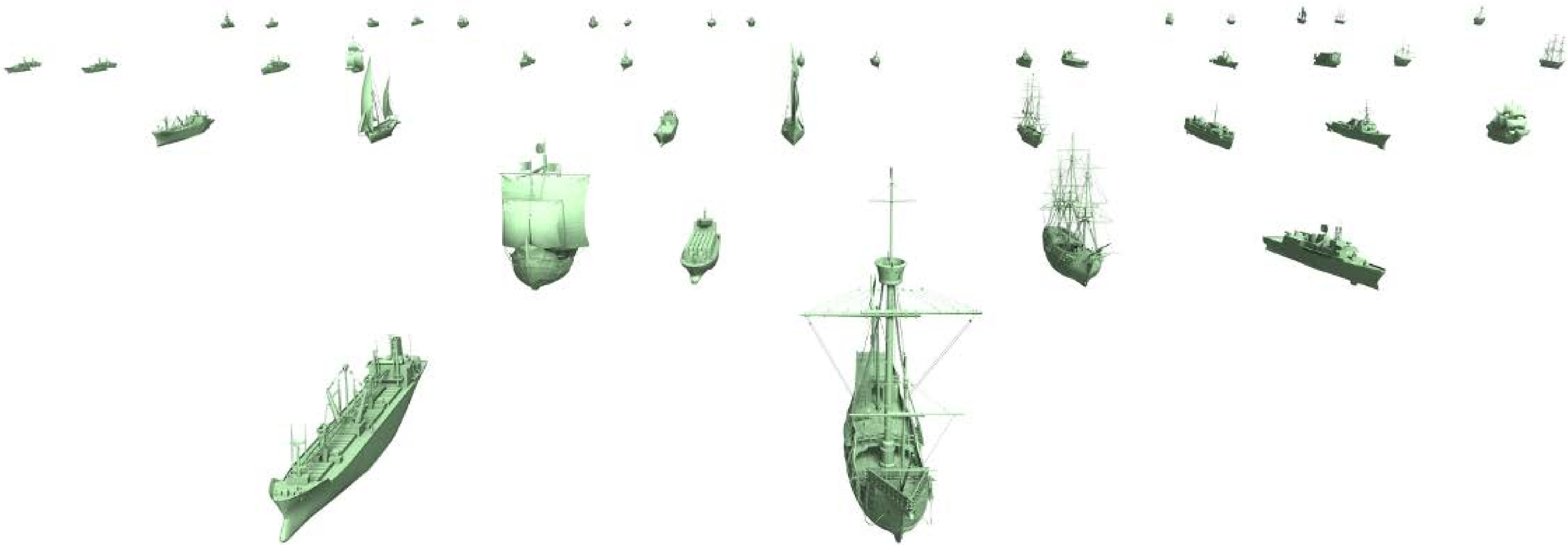
Meaningful parametrization: **Yes**

Data-driven: **Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

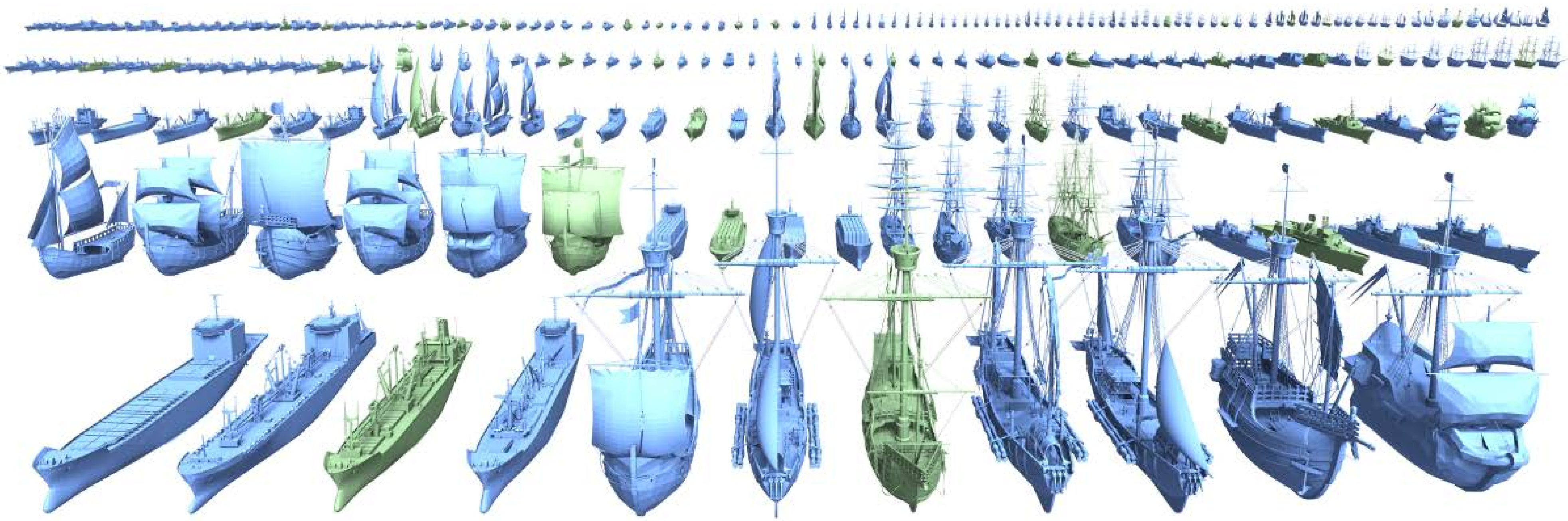


Generality: **Moderate**
Probabilistic: **Yes**
Meaningful parametrization: **Yes**
Data-driven: **Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

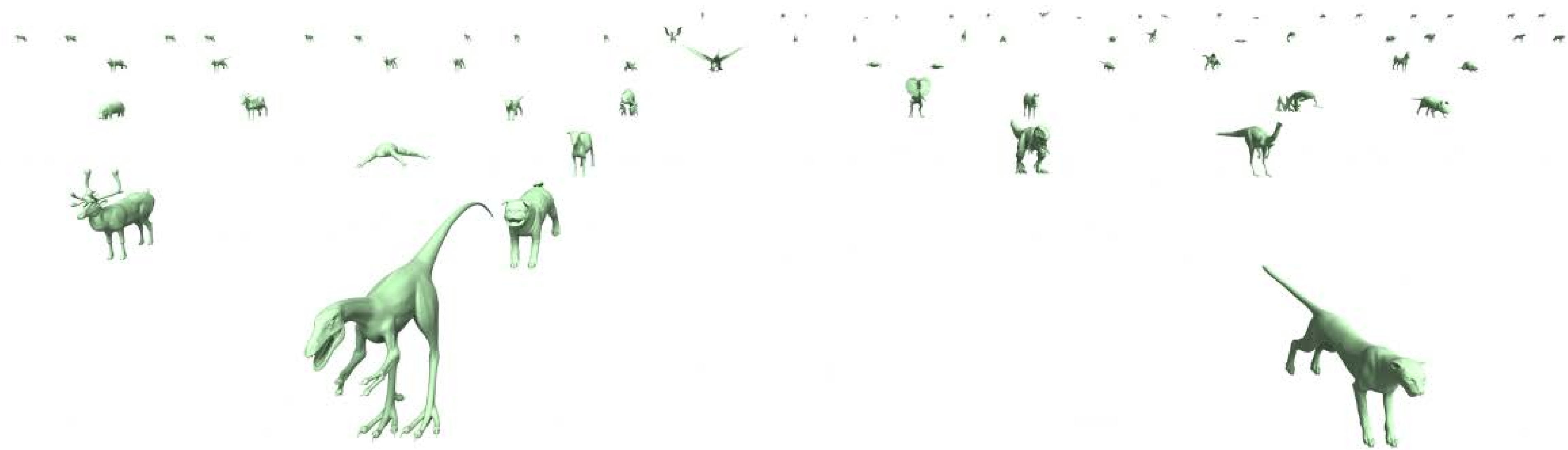


Generality:	Moderate	Meaningful parametrization:	Yes
Probabilistic:	Yes	Data-driven:	Yes

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

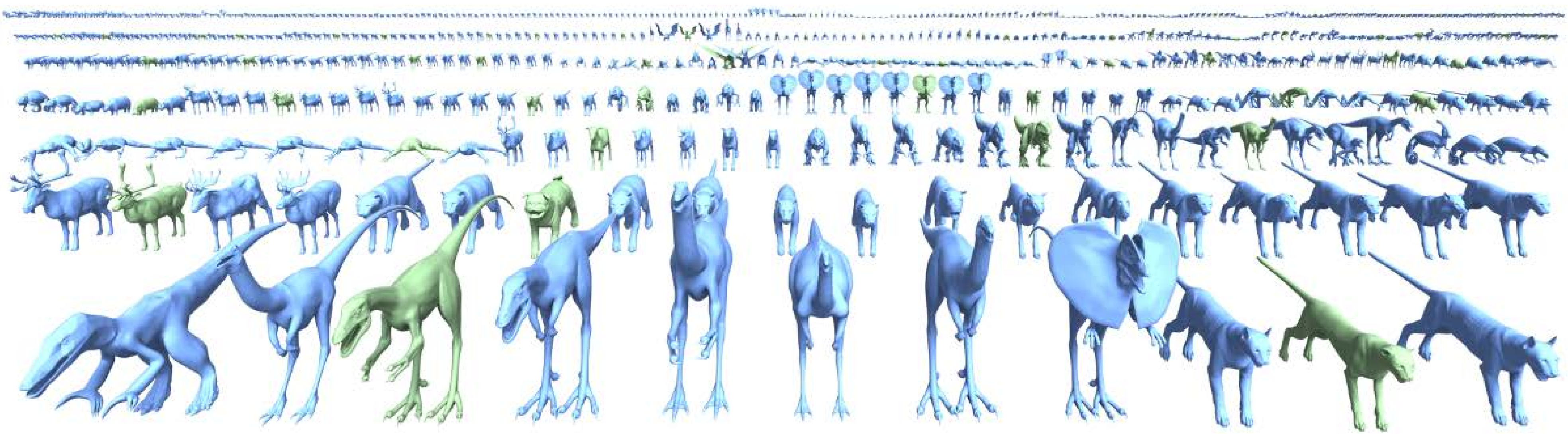


Generality:	**Moderate**	Meaningful parametrization:	**Yes**
Probabilistic:	**Yes**	Data-driven:	**Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: Probabilistic Assembly

(some assemblies are better than others)

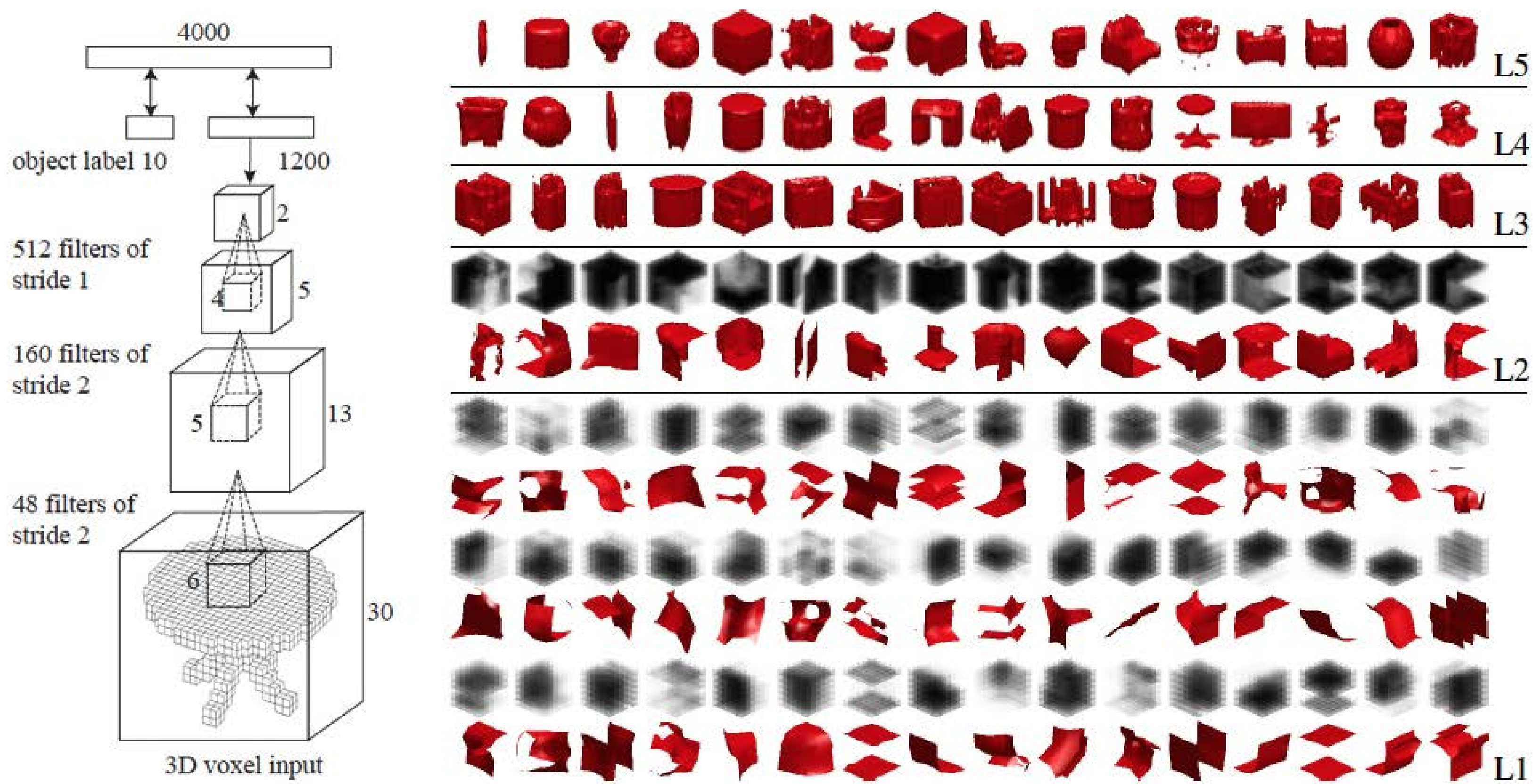


Generality:	**Moderate**	Meaningful parametrization:	**Yes**
Probabilistic:	**Yes**	Data-driven:	**Yes**

Kalogerakis, Chaudhuri, Koller and Koltun, 2012

Shape Space: 3D Deep Belief Network

(convolutional + fully-connected RBM, stacked layers)

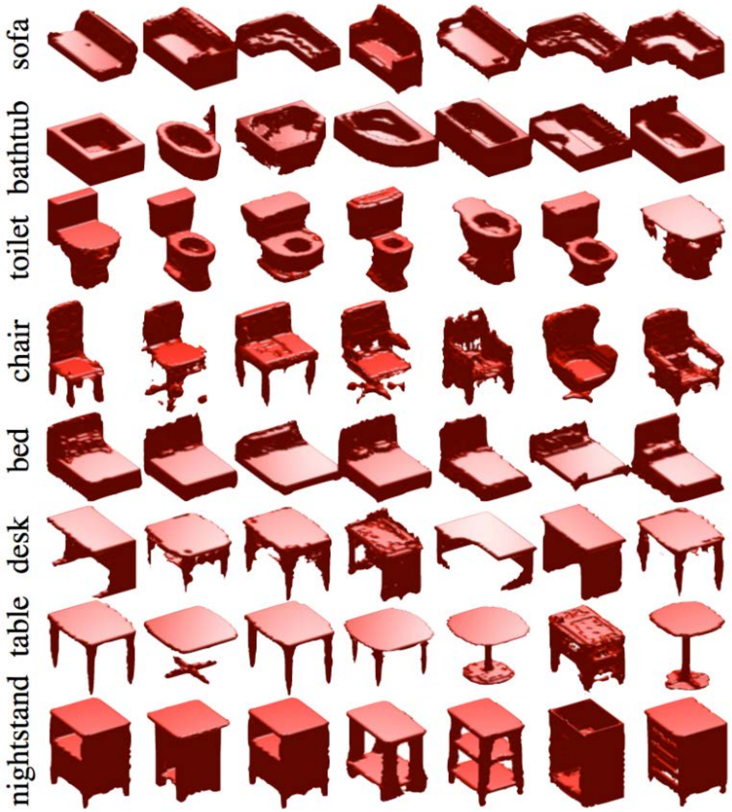


Generality: High
Probabilistic: Yes
Meaningful parametrization: No
Data-driven: Yes

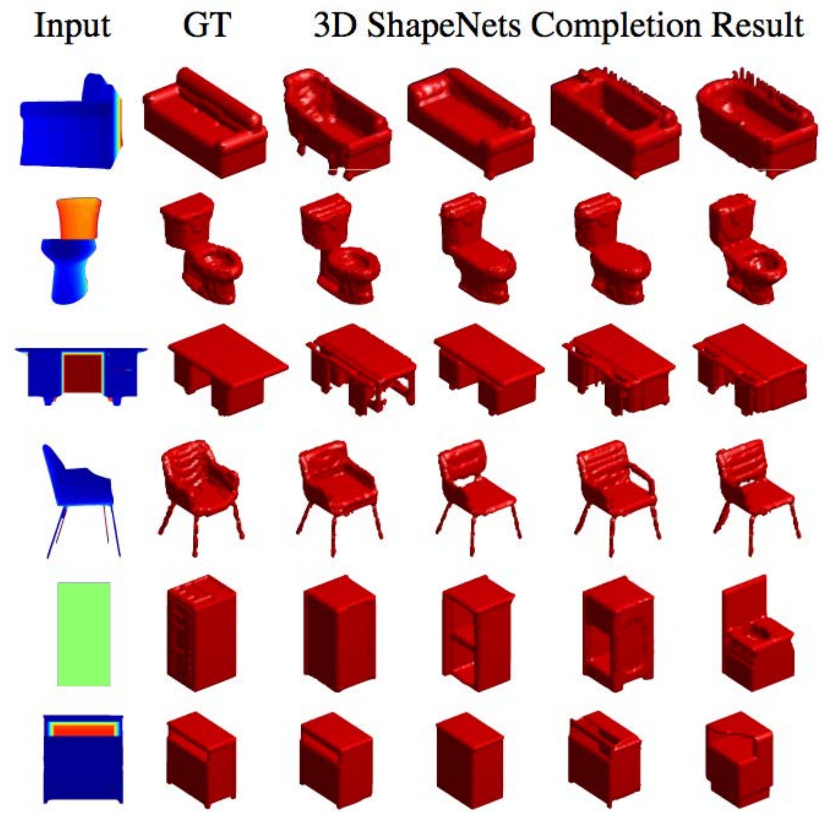
Wu, Song, Khosla, Yu, Zhang, Tang and Xiao, 2015

Shape Space: 3D Deep Belief Network

(convolutional + fully-connected RBM, stacked layers)



Sampled shapes



Completed shapes

Generality: **High**

Probabilistic: **Yes**

Meaningful parametrization: **No**

Data-driven: **Yes**

Wu, Song, Khosla, Yu, Zhang, Tang and Xiao, 2015

- Make a cute toy

- Make a cute toy

- Make an aerodynamic airplane

- Make a cute toy
- Make an aerodynamic airplane
- Make a comfortable chair

- Make a cute toy
- Make an aerodynamic airplane
- Make a comfortable chair
- Make an efficient bicycle

- Make a cute toy
- Make an aerodynamic airplane
- Make a comfortable chair
- Make an efficient bicycle
- Make a professional-looking webpage

- Make a cute **toy**
- Make an aerodynamic **airplane**
- Make a comfortable **chair**
- Make an efficient **bicycle**
- Make a professional-looking **webpage**

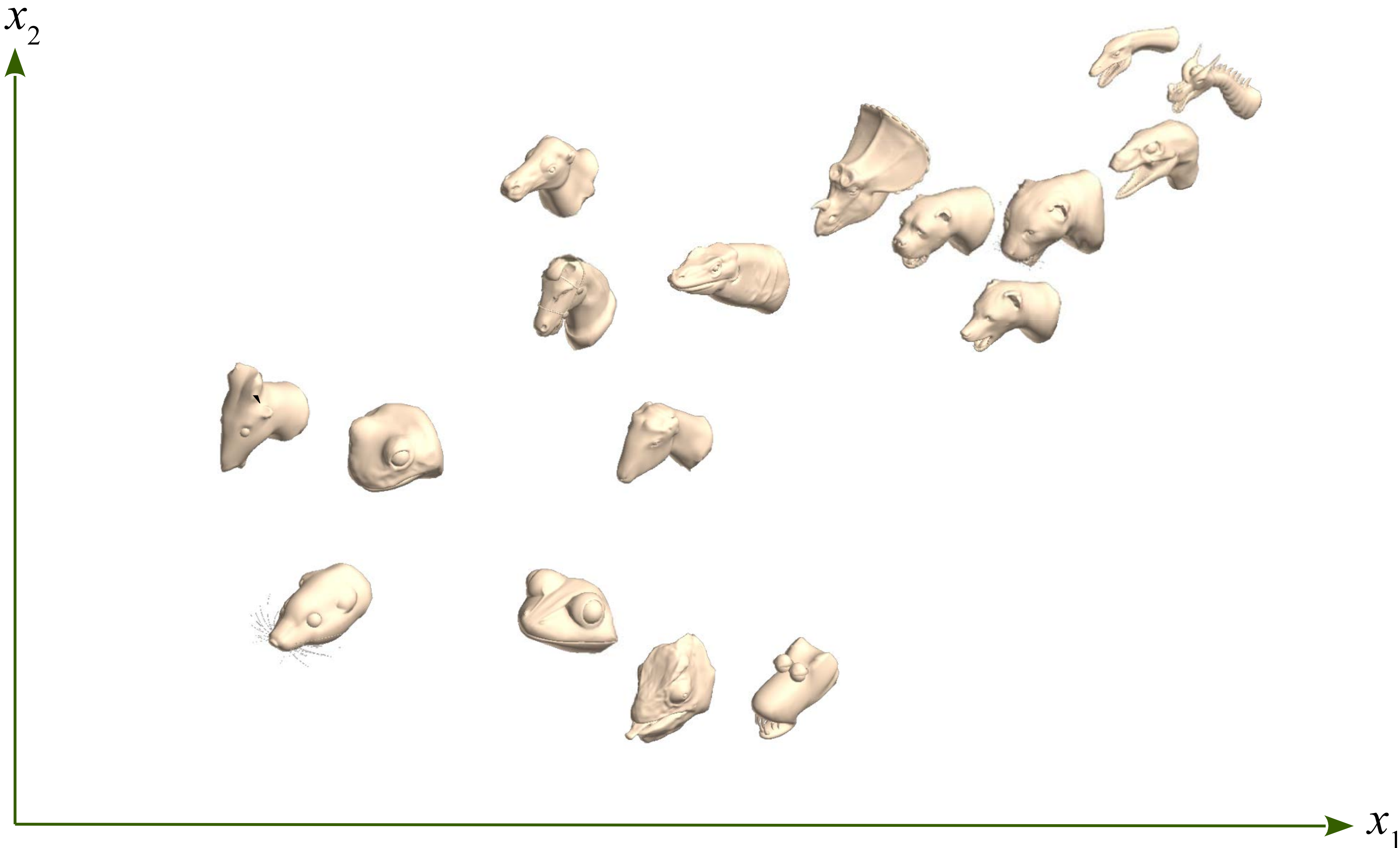
- Make a **cute** toy
- Make an **aerodynamic** airplane
- Make a **comfortable** chair
- Make an **efficient** bicycle
- Make a **professional**-looking webpage

Outline

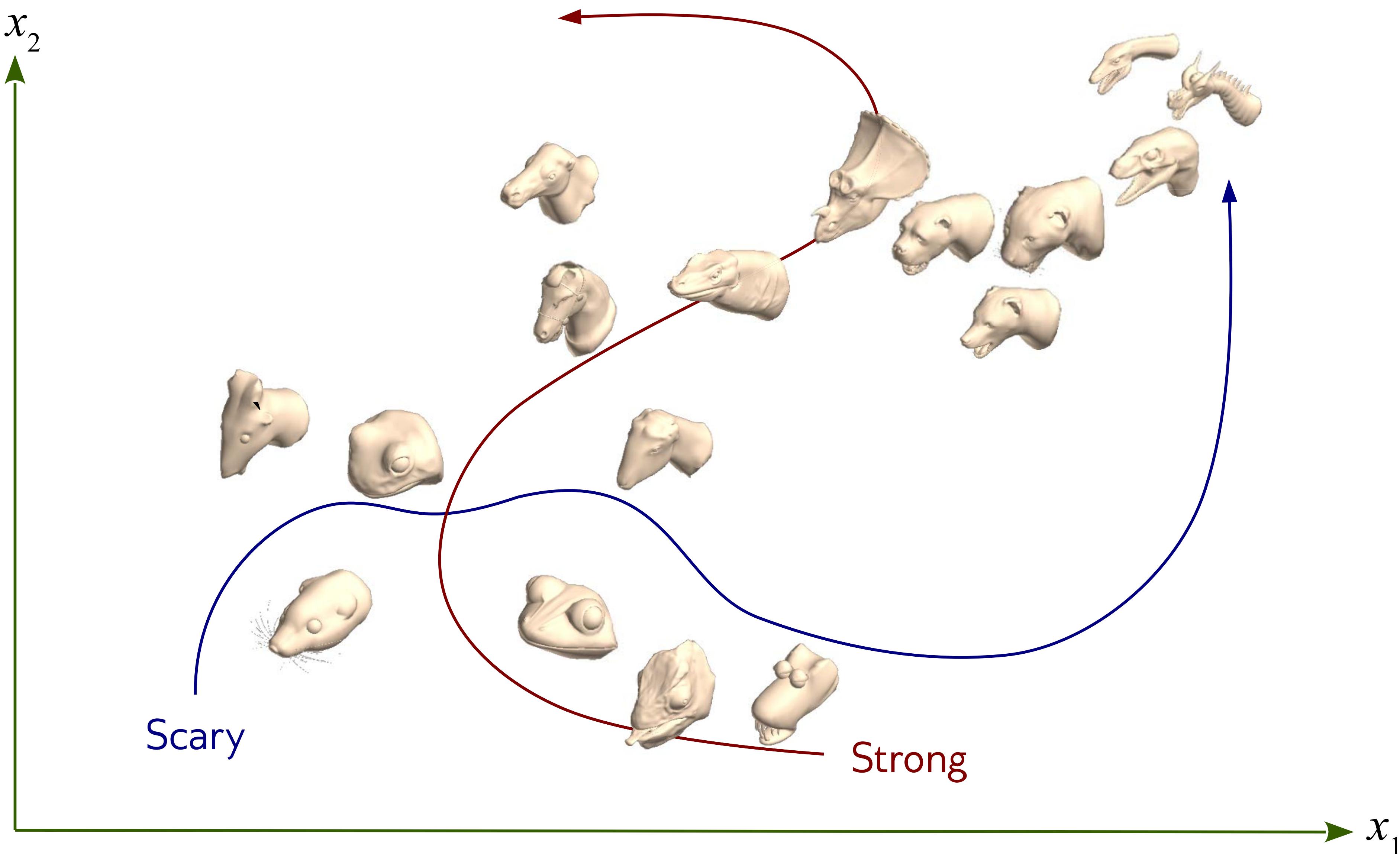
- Learning shape structure
  - **Probabilistic models** of shape

- Learning shape semantics
  - Semantic **attributes** (*scary, artistic, ...*)
  - Mechanical **function** (*this airplane should fly*...)
  - Human **interaction** (*sit comfortably in a chair...*)

Semantic Basis for Shape Space



Semantic Basis for Shape Space



A cute toy for a small child

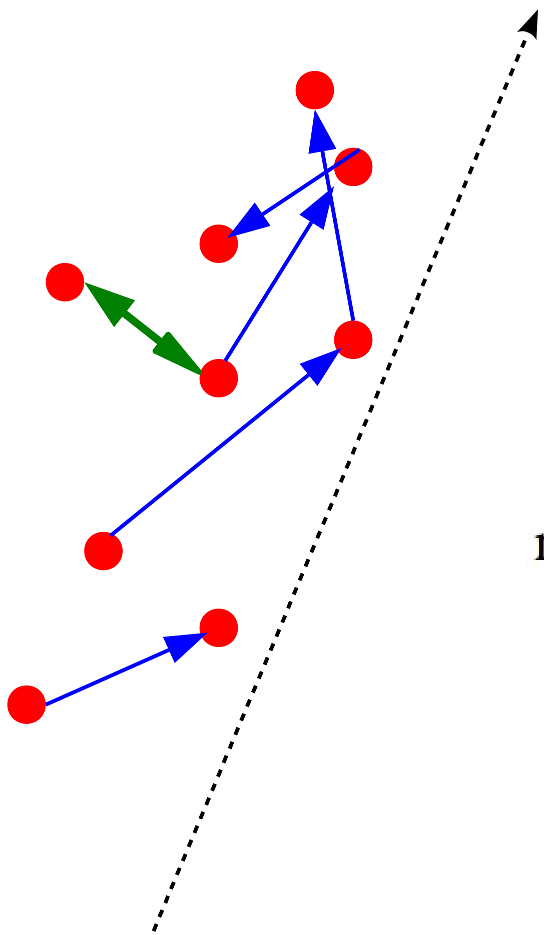
Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

Learning Semantic Attributes

- Crowdsource **comparative adjectives**
  - Amazon Mechanical Turk
  - Schelling survey
- Crowdsource comparisons for **training pairs**
  - A is more [......] than B
- Learn **ranking functions**
  - f: shape features $\rightarrow \mathbb{R}$
  - Rank-SVM with transformed features & sigmoid loss
  - Iterate with cross-correlation between attributes
  - Extend to multi-component rankings

Learning Semantic Attributes

- Rank-SVM: Project features onto linear subspace that best preserves pairwise orderings



Learn $r_m(\mathbf{x}) = \mathbf{w}_m \cdot \mathbf{x}$

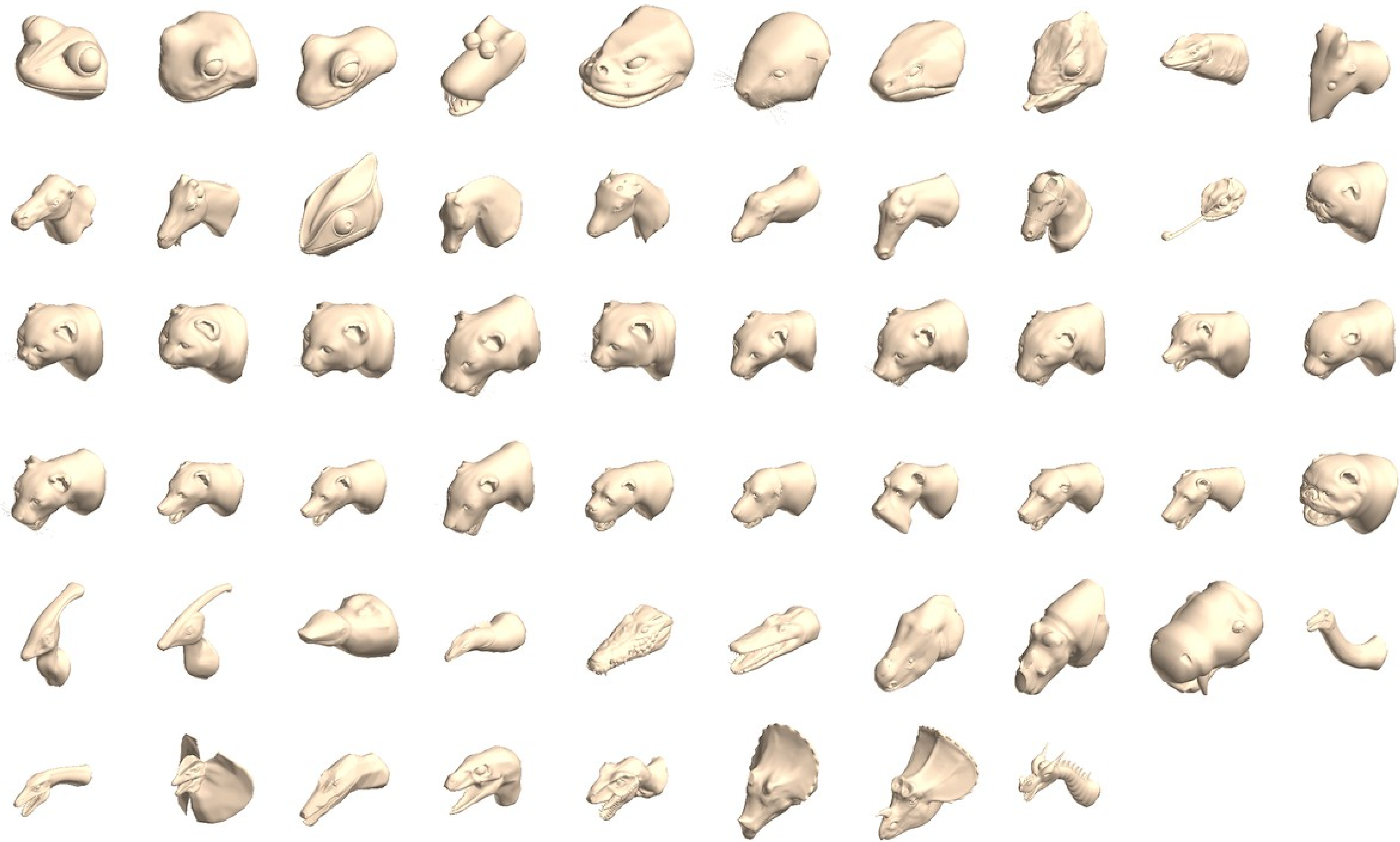
s.t.

$$\forall (i,j) \in O_m : \ \mathbf{w}_m \cdot \mathbf{x}_i > \mathbf{w}_m \cdot \mathbf{x}_j$$
$$\forall (i,j) \in S_m : \ \mathbf{w}_m \cdot \mathbf{x}_i = \mathbf{w}_m \cdot \mathbf{x}_j$$

$$\text{minimize } \ \|\mathbf{w}_m\|_2^2 + \mu \sum_{i,j \in O_m} c_{ij}(1 - \sigma(\mathbf{w}_m(\mathbf{x}_i - \mathbf{x}_j))) + \nu \sum_{i,j \in S_m} c_{ij}\sigma(|\mathbf{w}_m(\mathbf{x}_i - \mathbf{x}_j)|)$$

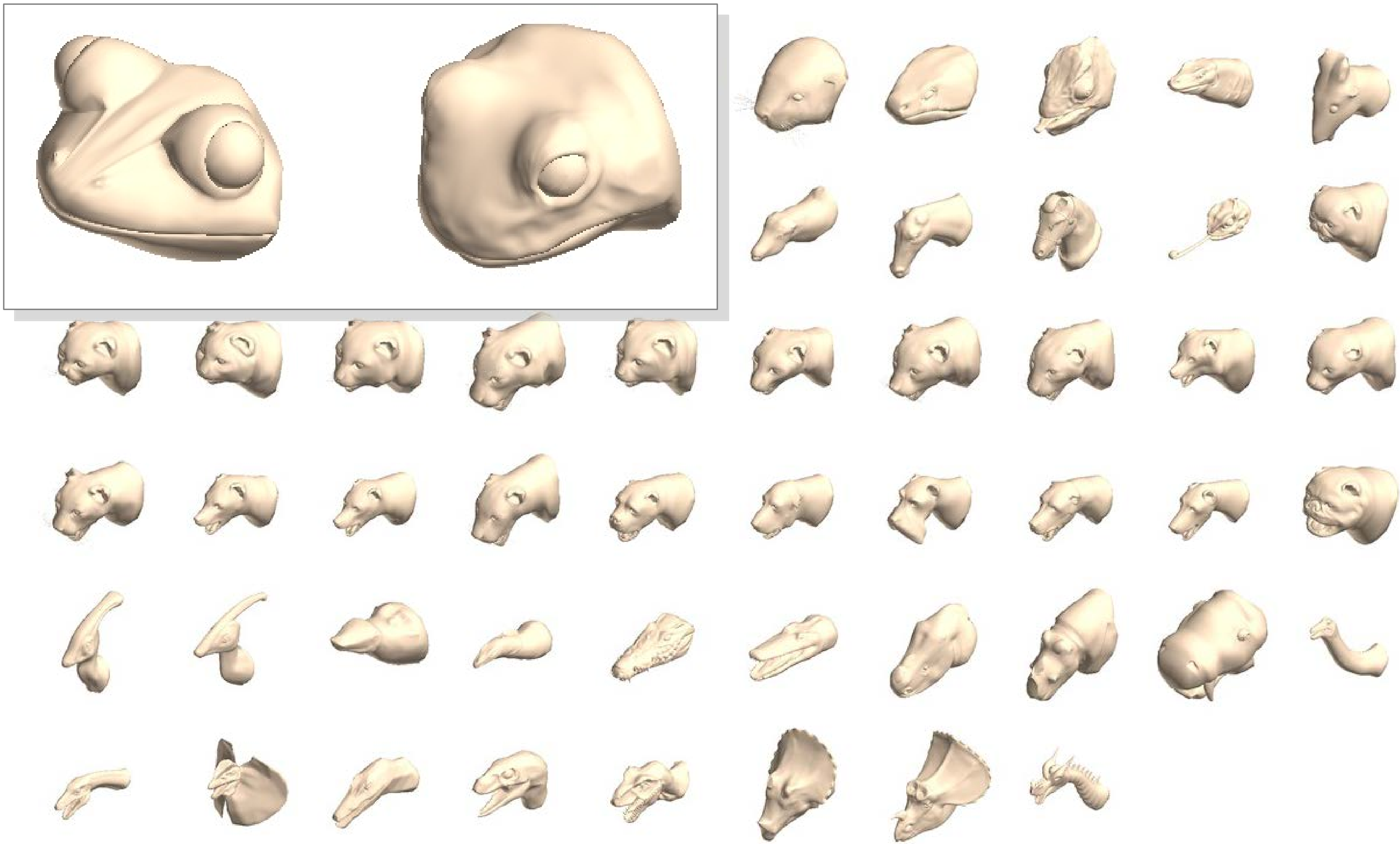
Chapelle 2007, Parikh and Grauman 2011, Chaudhuri et al. 2013

“Dangerous”



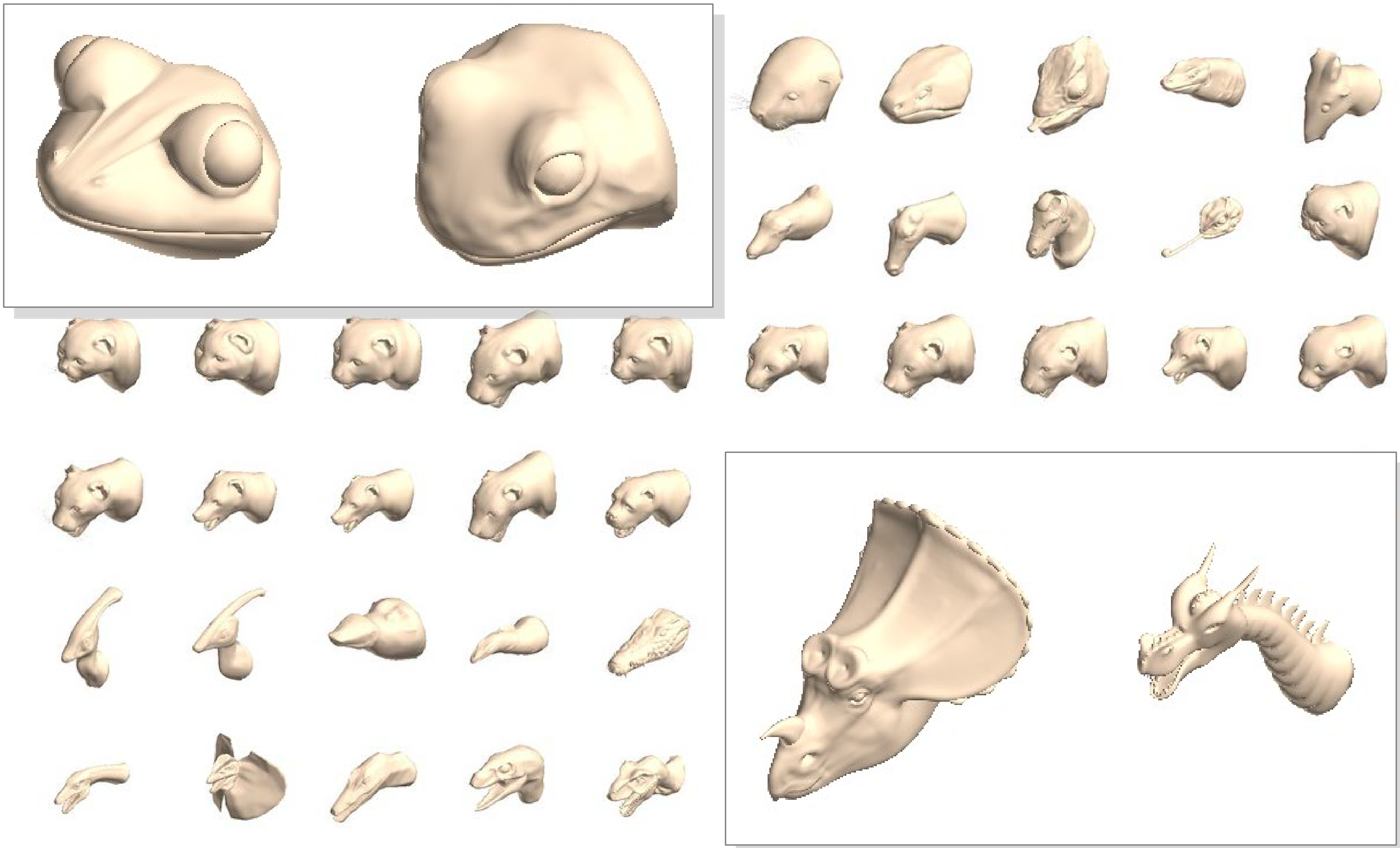
Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

“Dangerous”



Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

“Dangerous”

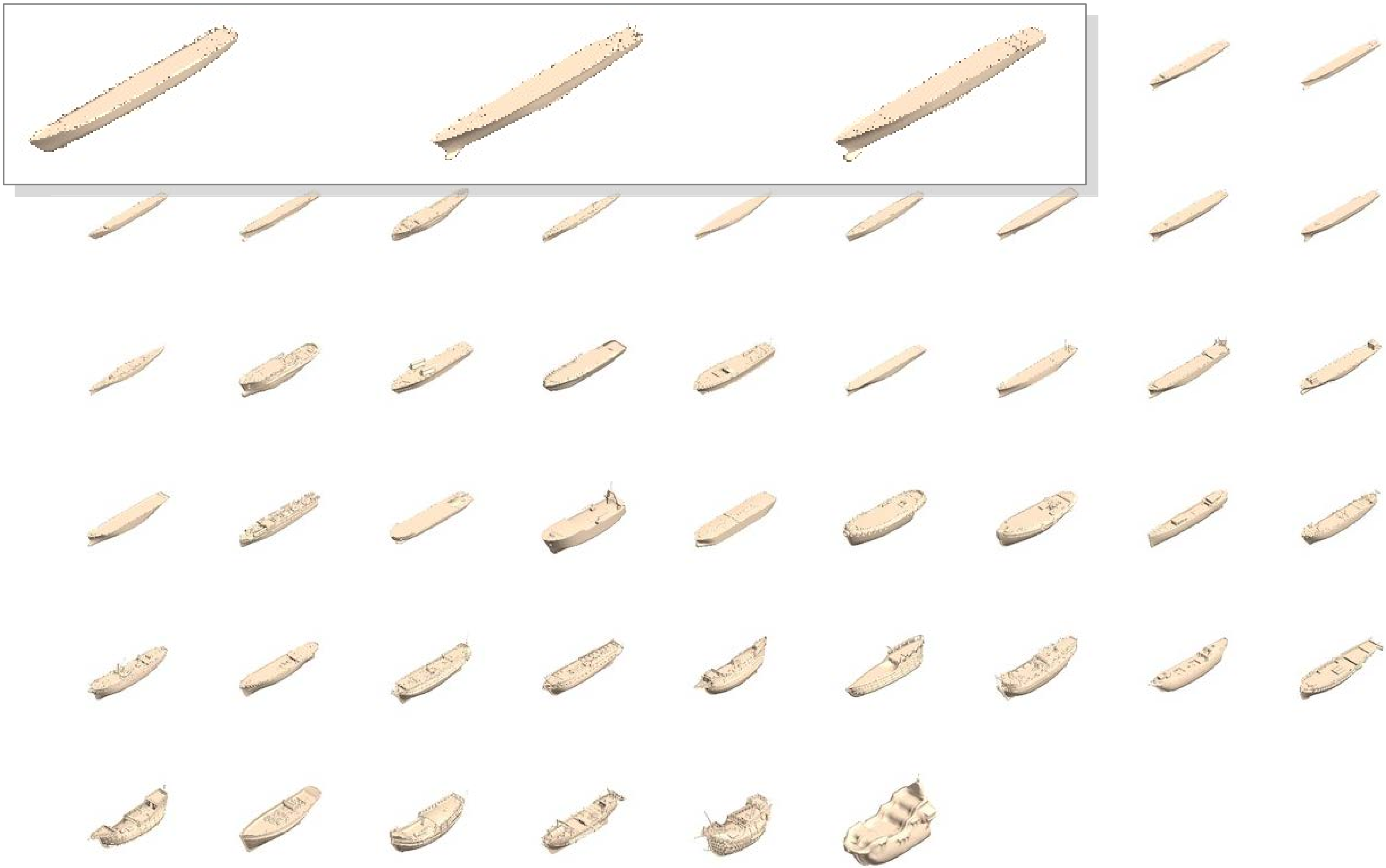


Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

“Old-fashioned”

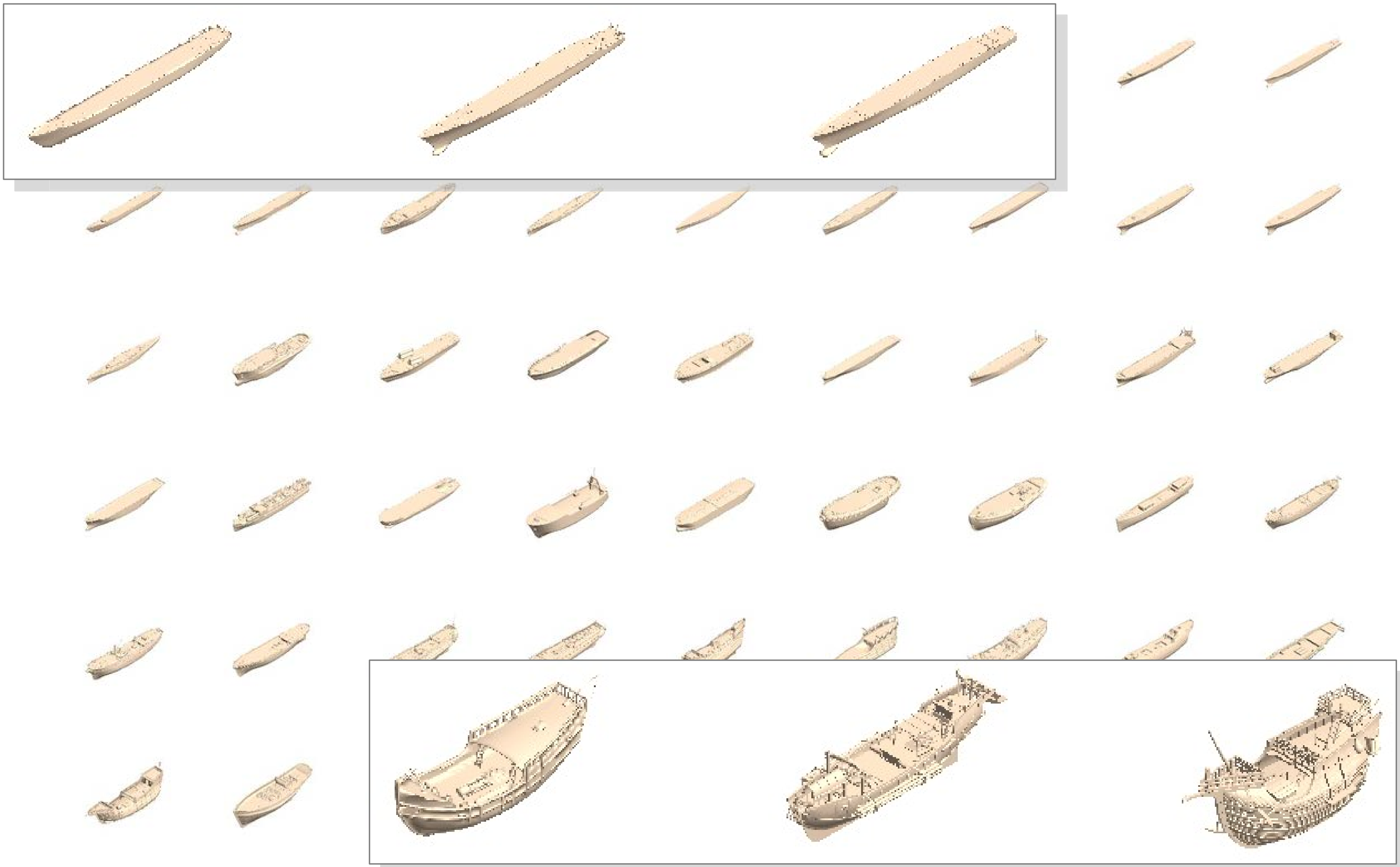
Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

“Old-fashioned”



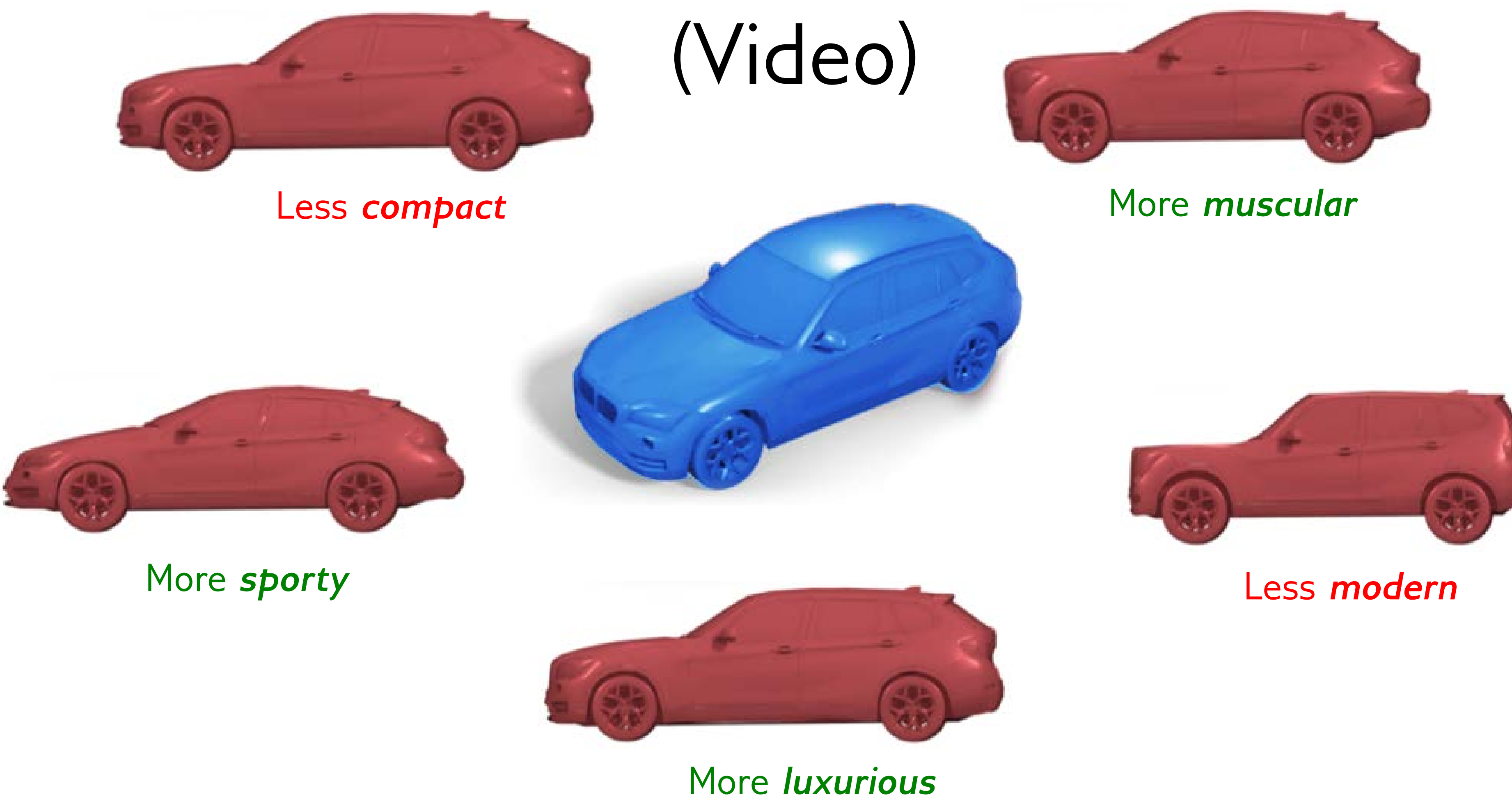
Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

“Old-fashioned”



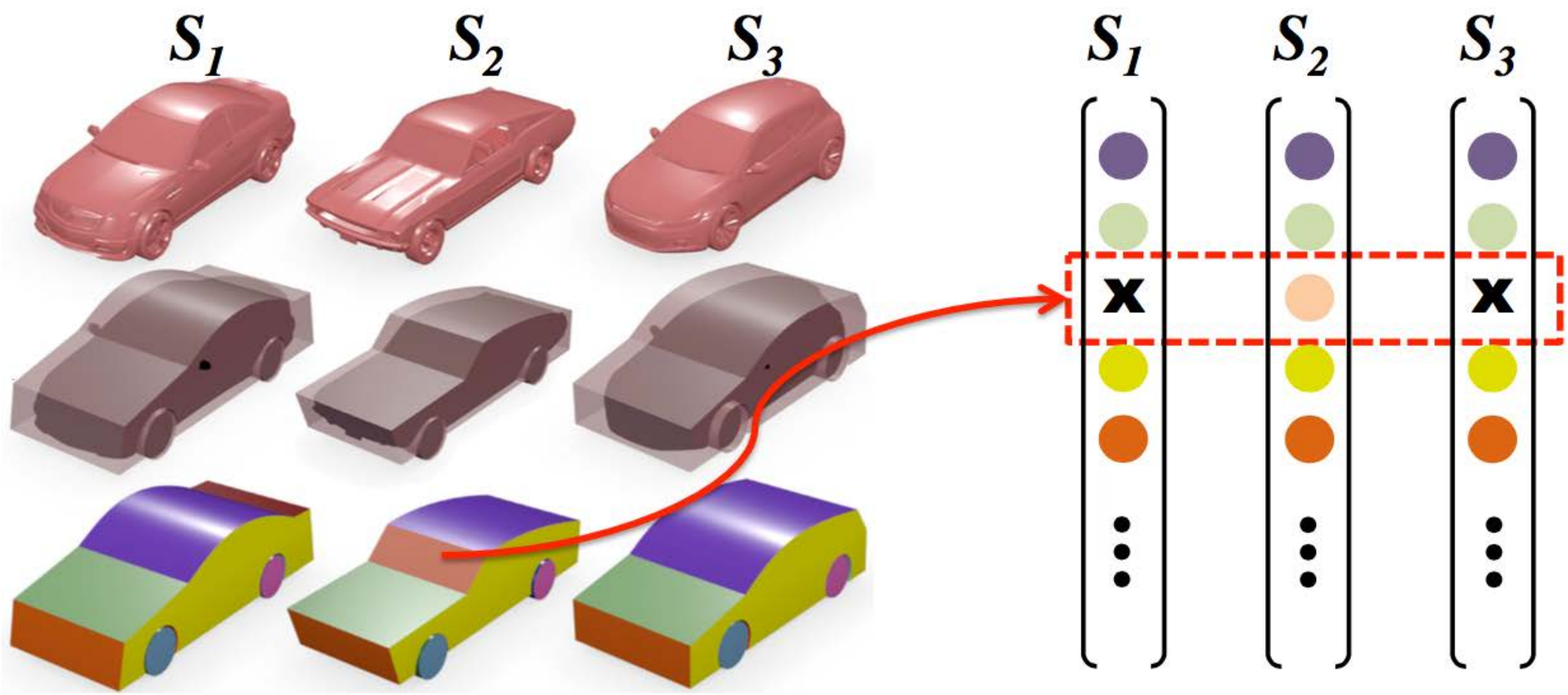
Chaudhuri, Kalogerakis, Giguere and Funkhouser, 2013

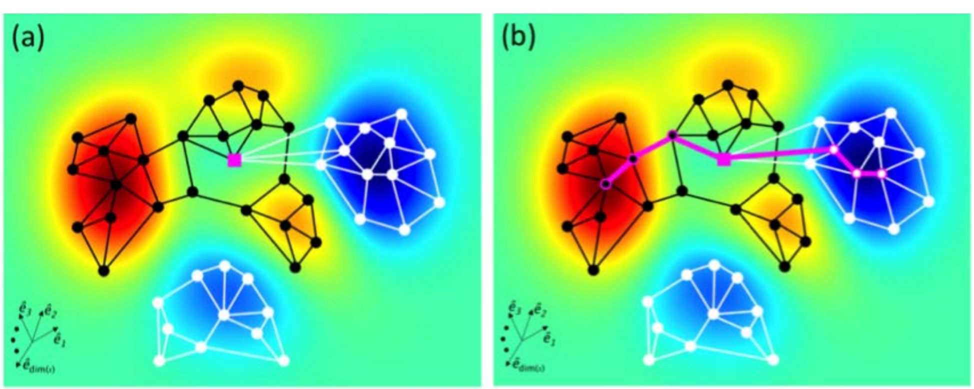
Semantic Shape Editing



Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Semantic Shape Editing



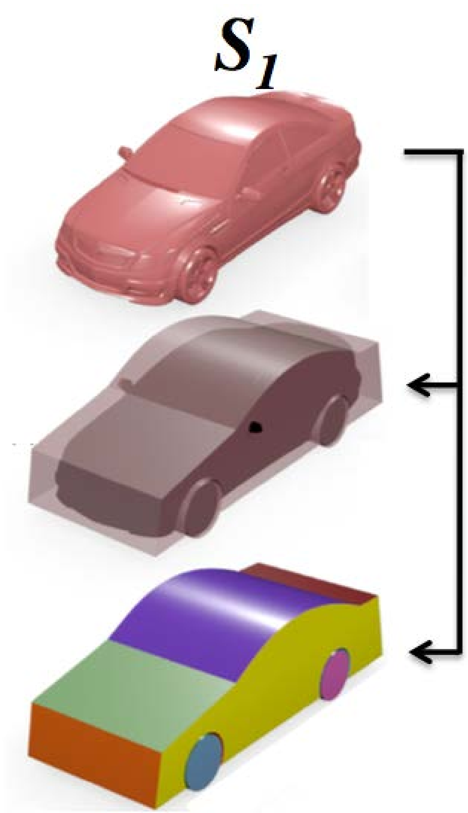


$$\underset{p_i}{\text{minimize}} \sum_{\{\mathcal{V}_h,\mathcal{E}_h\}\in\mathcal{H}} \left(\sum_{i\in\mathcal{V}_h} |p_i - f_i| + \lambda \sum_{j\in\mathcal{E}_h} -log\left(\frac{\beta_j}{\pi}\right)\right)$$

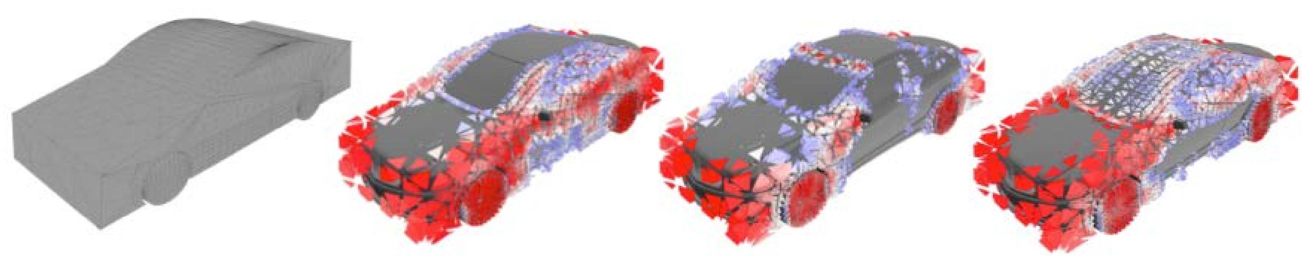
Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Deformation Space = Feature Space

Use deformation handle parameters as shape features



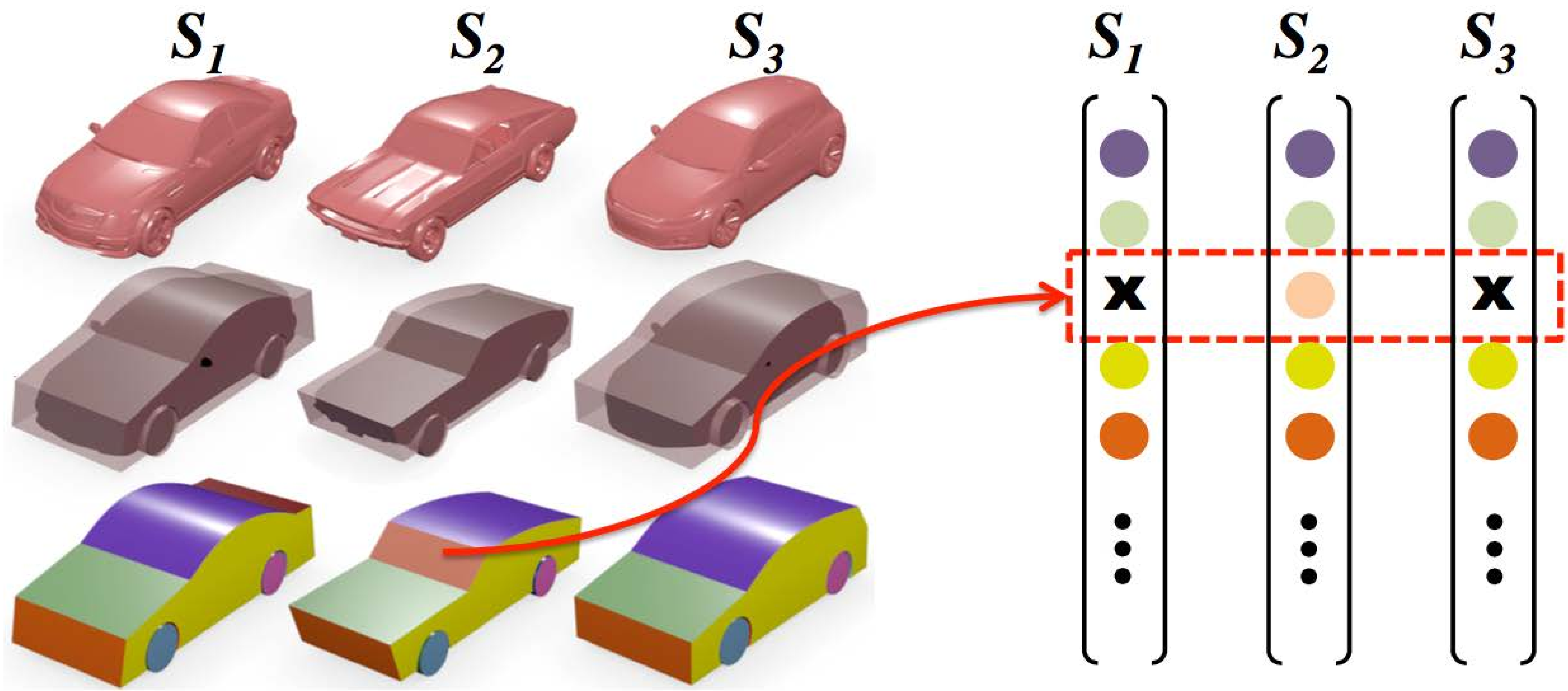
- Sphere: $r, p_i | i \in \{x, y, z\}$
- Cylinder: $r, p_i, \theta_i | i \in \{x, y, z\}$
- Circular Cone: $r, \beta, p_i, \theta_i | i \in \{x, y, z\}$
- Quadric: $k_1, k_2, p_i, \theta_i | i \in \{x, y, z\}$



Yumer and Kara, SIGGRAPH Asia 2014

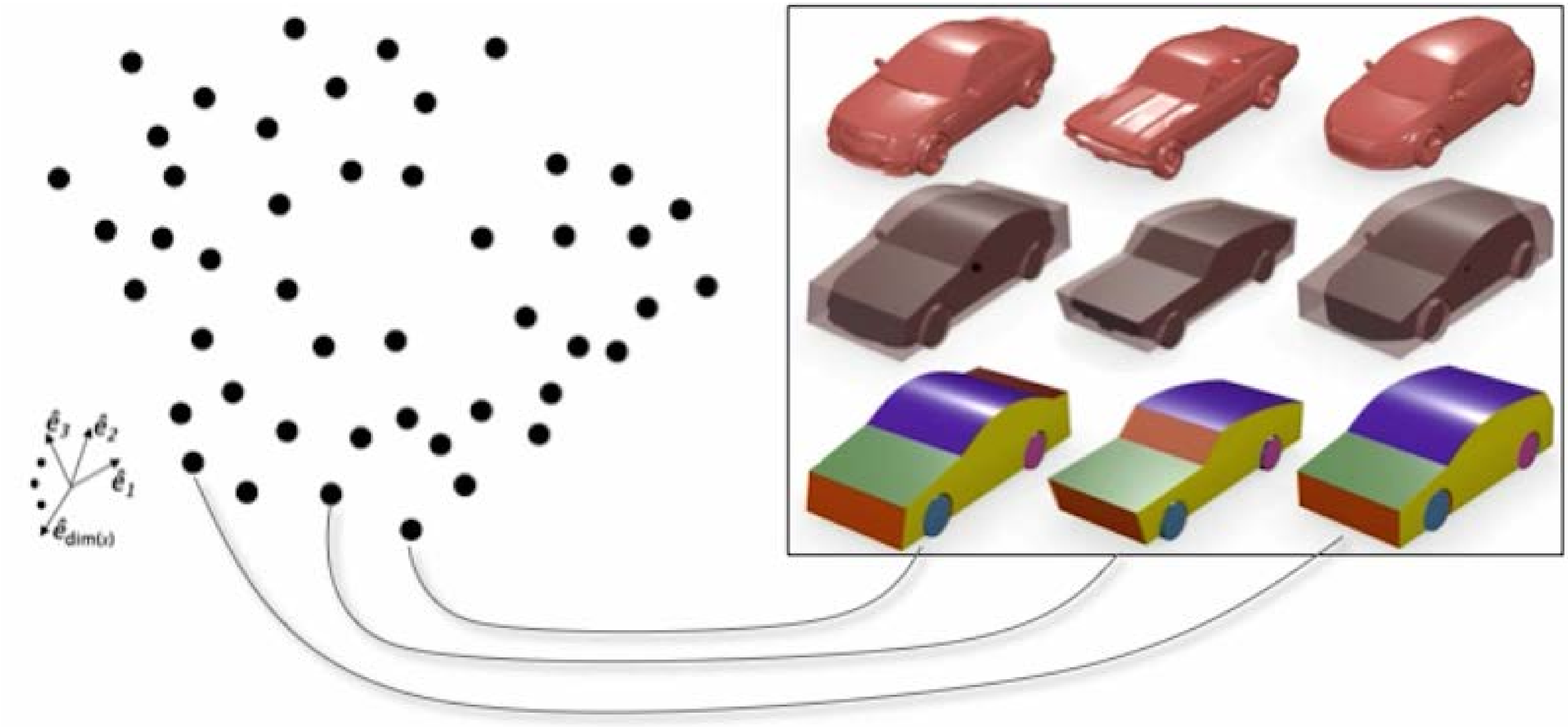
Deformation Space = Feature Space

Use deformation handle parameters as shape features



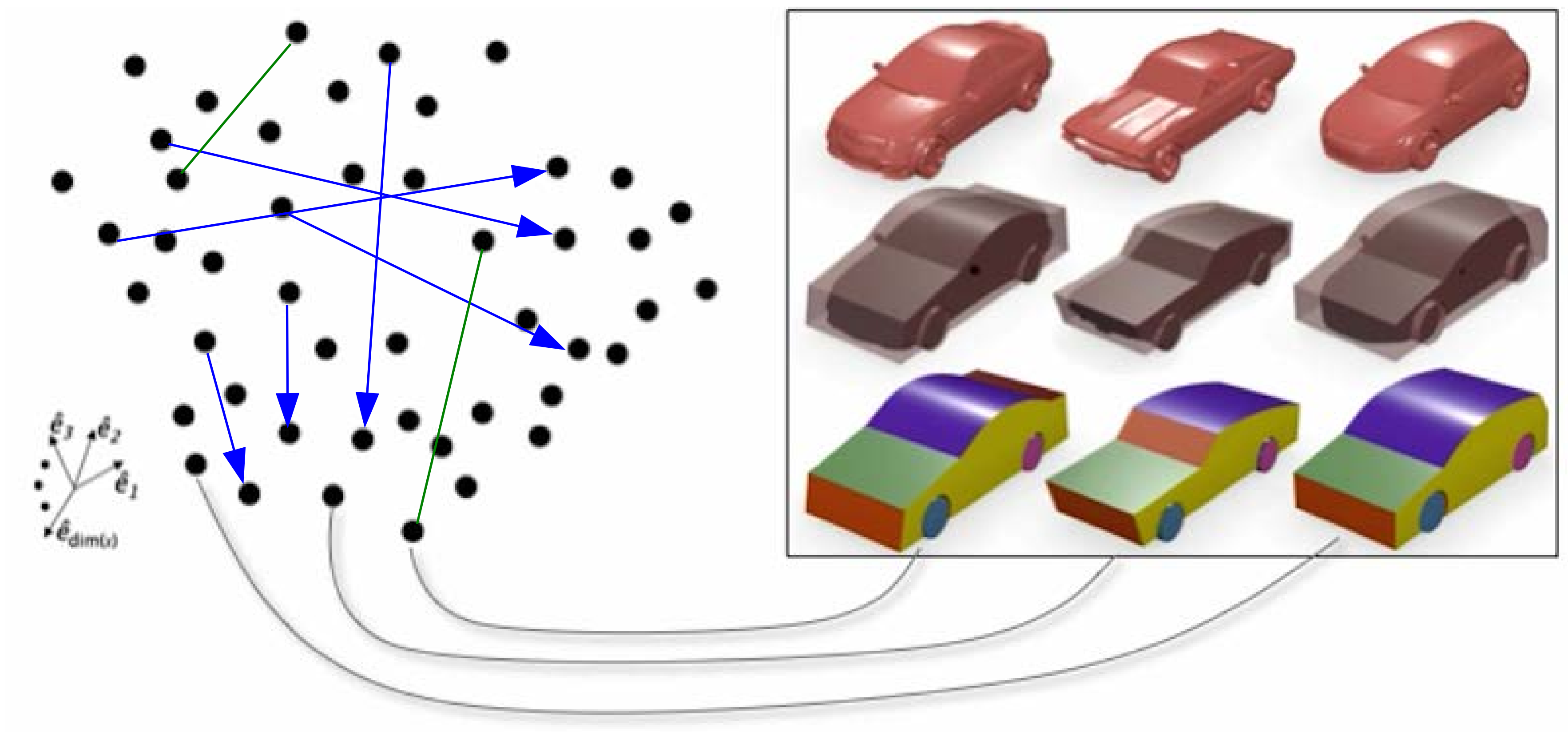
Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Deformation Space = Feature Space



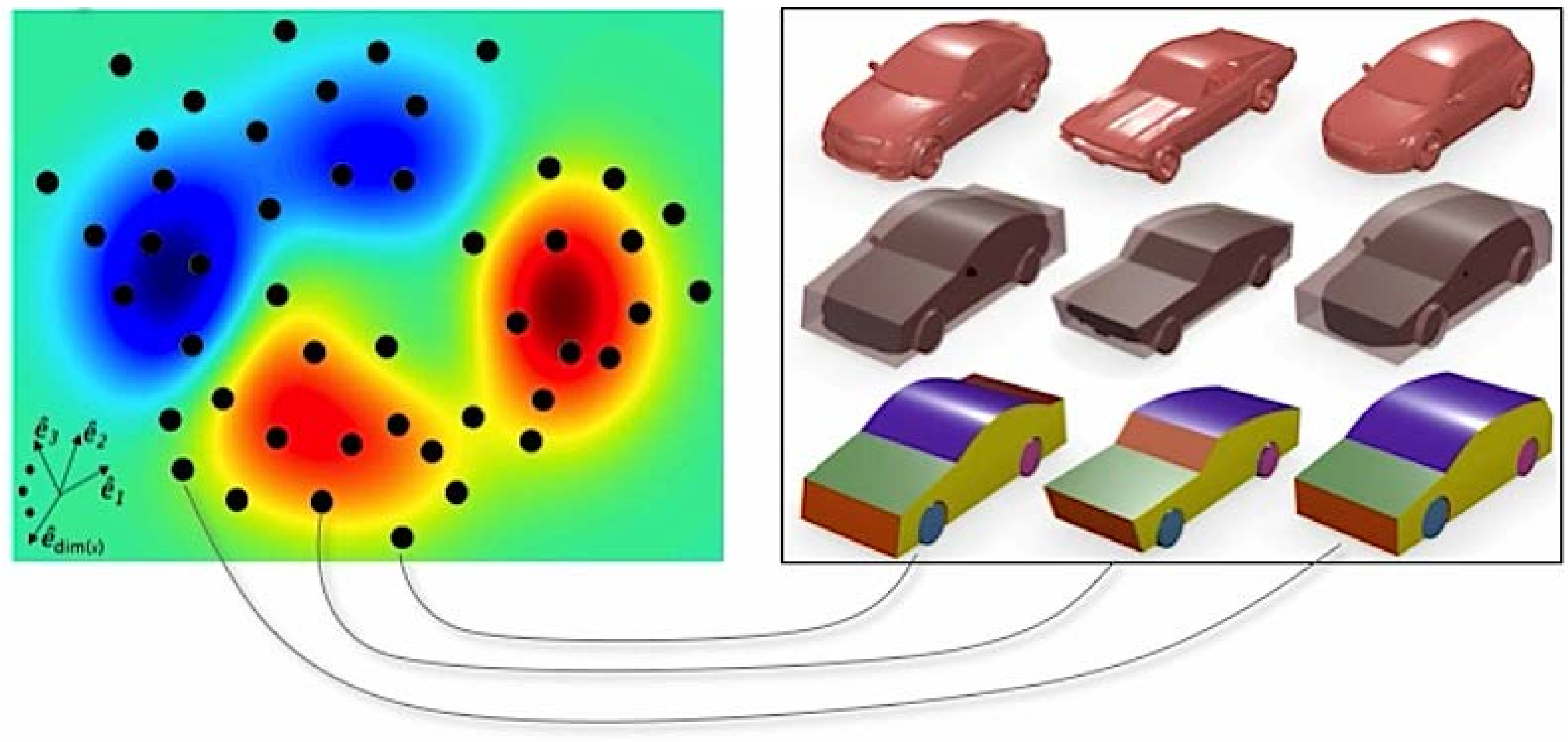
Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Comparisons in Feature Space



Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Attribute Distribution over Feature Space



Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Attribute Learning: Absolute Scores

- Assume normally distributed absolute scores

$$ {}_aP_i \sim N({}_a\mu_i, {}_a\sigma_i^2) = \frac{1}{{}_a\sigma_i\sqrt{2\pi}} e^{-\frac{(x - {}_a\mu_i)^2}{2\,{}_a\sigma_i^2}} $$

- Pairwise comparisons modeled as difference of normal distributions

from user study statistics

$$ {}_aP_i - {}_aP_j \sim N({}_a\mu_{ij}, {}_a\sigma_{ij}^2) = N({}_a\mu_i - {}_a\mu_j, {}_a\sigma_i^2 + {}_a\sigma_j^2) $$

- Solve overdetermined linear system

$$ {}_a\mu_i - {}_a\mu_j = {}_a\mu_{ij} \qquad {}_a\sigma_i^2 + {}_a\sigma_j^2 = {}_a\sigma_{ij}^2 $$

Attribute Learning: Scoring Function

$$\widetilde{f}_a(\mathbf{x}_s) = \sum_{t \in \mathcal{T}} \frac{w_t(\mathbf{x}_s)}{\sum_j w_j(\mathbf{x}_s)} f_a(\mathbf{x}_t)$$

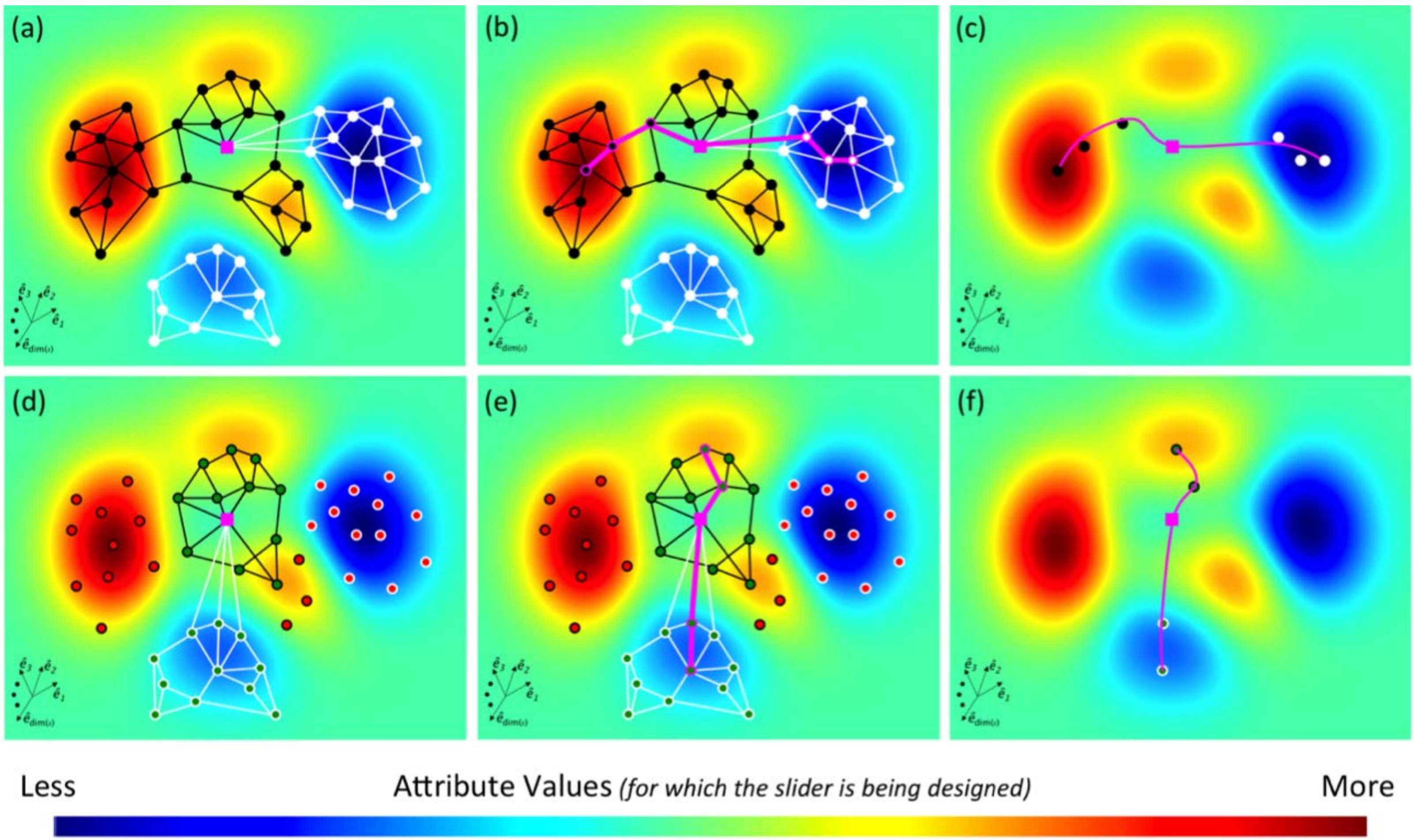
$$w_t(\mathbf{x}_s) = {}_a r_t \| \mathbb{1}_s \cdot \mathbb{1}_t \cdot (\mathbf{x}_s - \mathbf{x}_t) \|^{-p}$$

- $\mathbf{x}_s$: feature vector of the new shape
- $\mathbf{x}_t$: feature vector of shape ***t*** from database
- $\widetilde{f}_a(\mathbf{x}_s)$: attribute score of the new shape
- $f_a(\mathbf{x}_t)$: attribute score of shape ***t*** from database
- $\mathcal{T}$: set of all shapes in the database
- $\mathbb{1}_i$: indicator function of feature vector ***i***
- ${}_a r_t$: reliability factor

$$f_a(\mathbf{x}_t) = {}_a\mu_t$$

$${}_a r_t = 1 / {}_a\sigma_k^2$$

Constrained Path Traversal

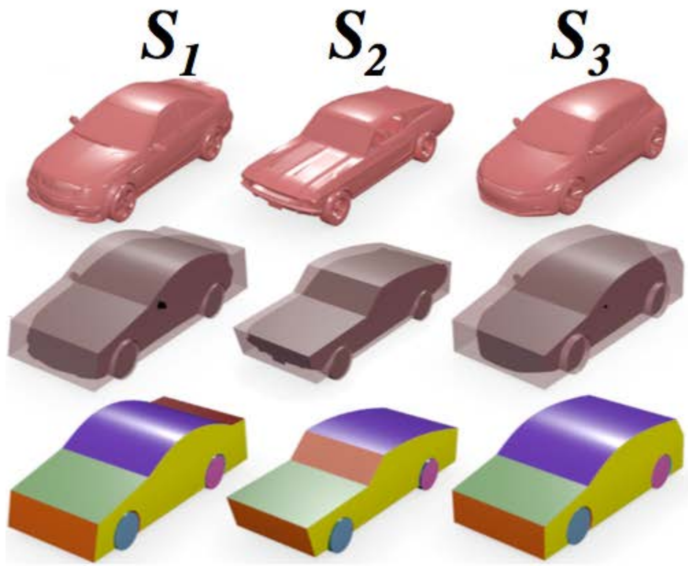


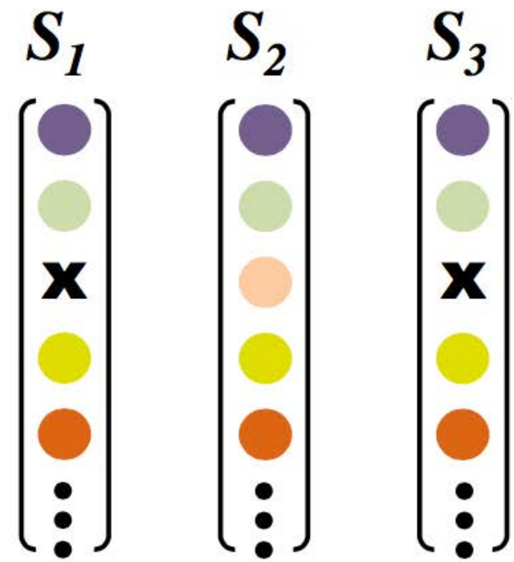
■ : Edited shape's current location in feature space
● : Shapes with higher attribute value
○ : Shapes with lower attribute value
— : Edited shape's spline path mapped to the slider
●● : Shapes that violate the active constraint
●● : Shapes that do not violate the active constraint

Yumer, Chaudhuri, Hodgins and Kara, SIGGRAPH 2015

Deformation from a Given Feature Vector

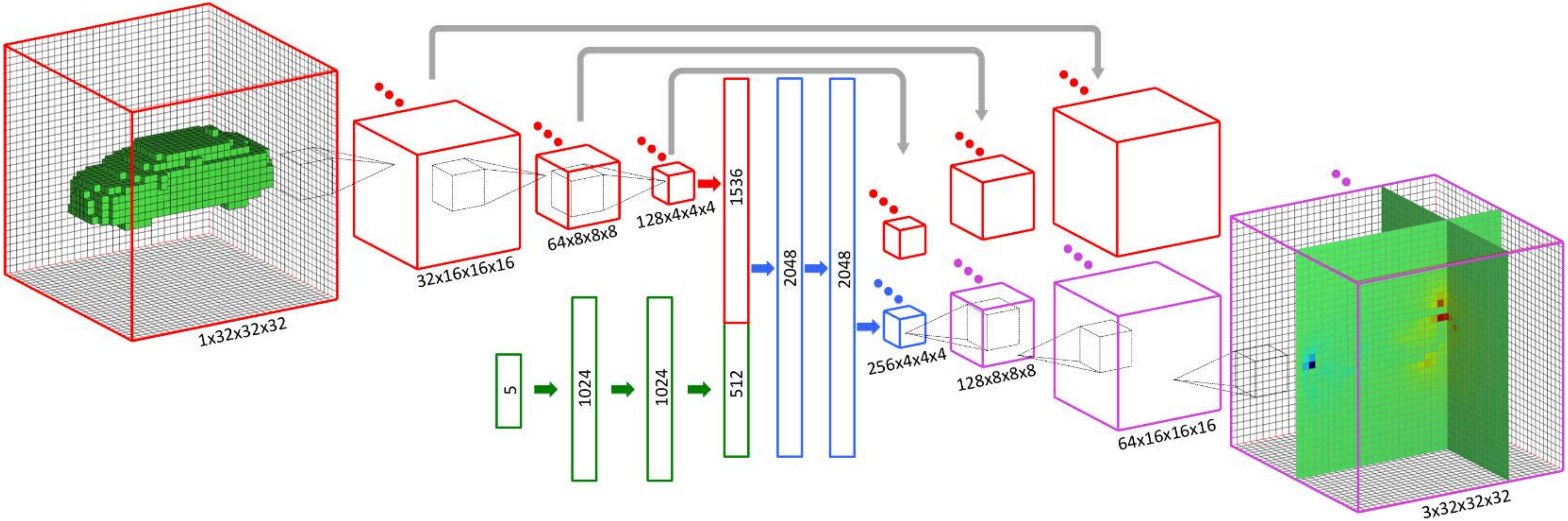
Flashback: Deformation handle parameters = shape feature vector

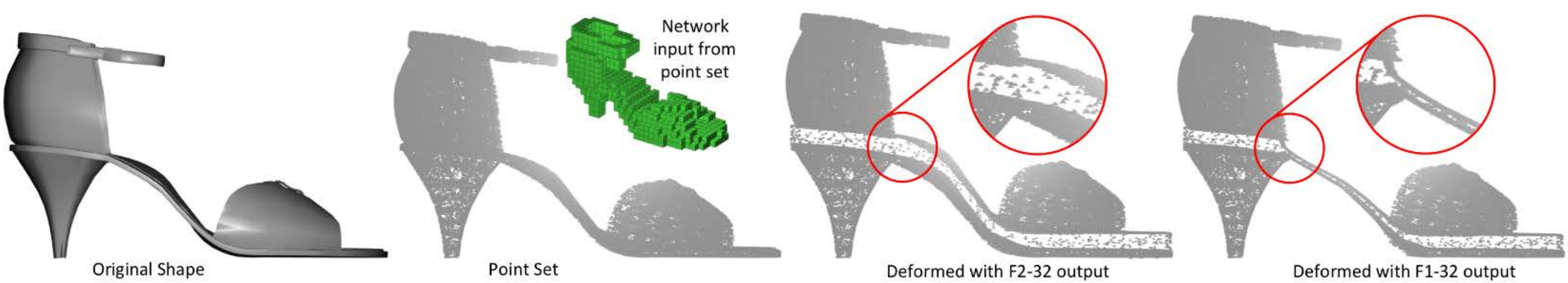




$$\underset{p_i}{\text{minimize}} \quad \sum_{\{\mathcal{V}_h, \mathcal{E}_h\} \in \mathcal{H}} \left(\sum_{i \in \mathcal{V}_h} |p_i - f_i| + \lambda \sum_{j \in \mathcal{E}_h} -log\left(\frac{\beta_j}{\pi}\right) \right)$$

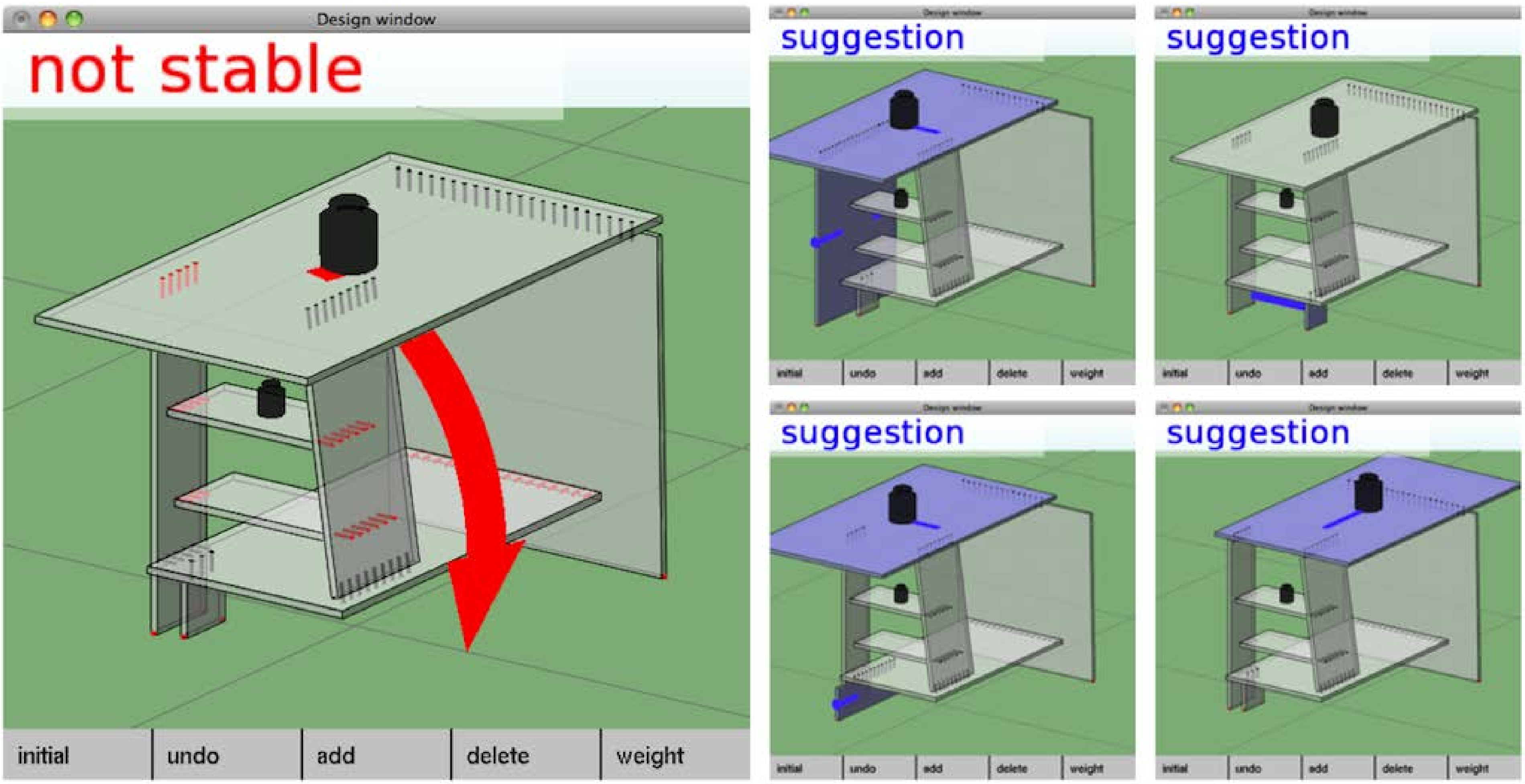
Semantic Deformation Flow





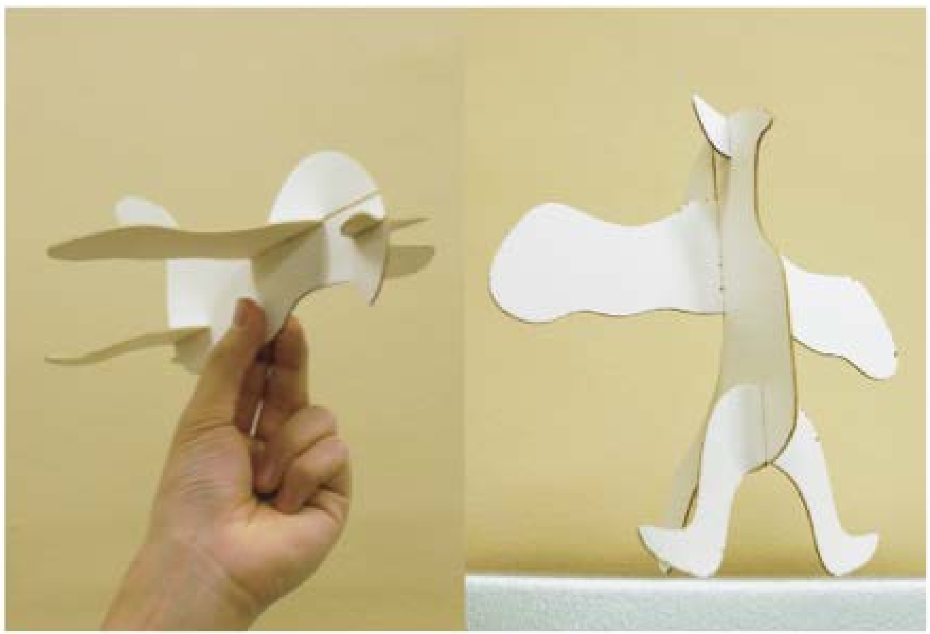
Yumer and Mitra, ECCV 2016

Designing for Mechanical Function



Umetani, Igarashi and Mitra, 2012

Designing for Mechanical Function

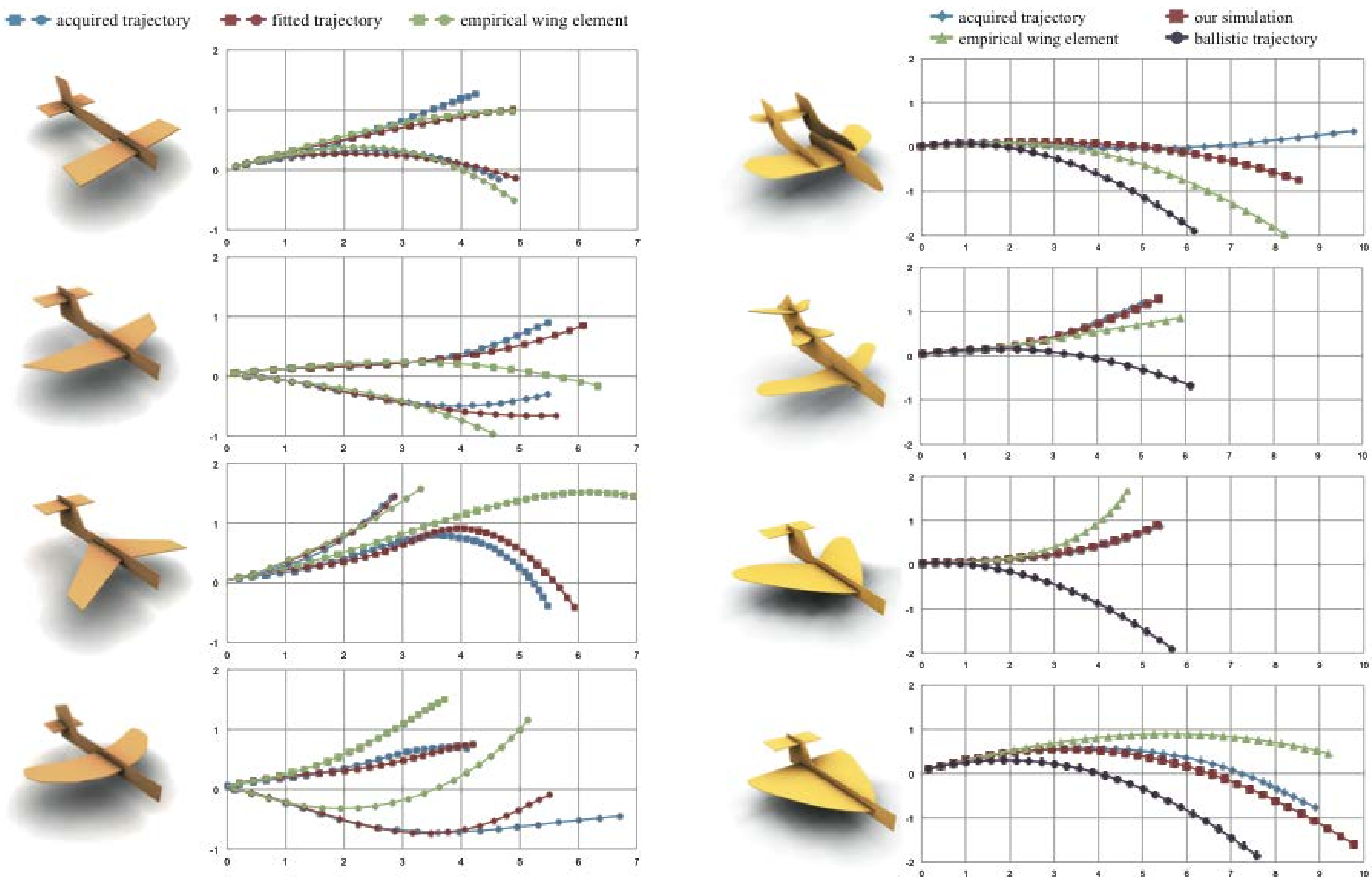


Umetani, Koyama, Schmidt and Igarashi, 2014

Designing for Mechanical Function

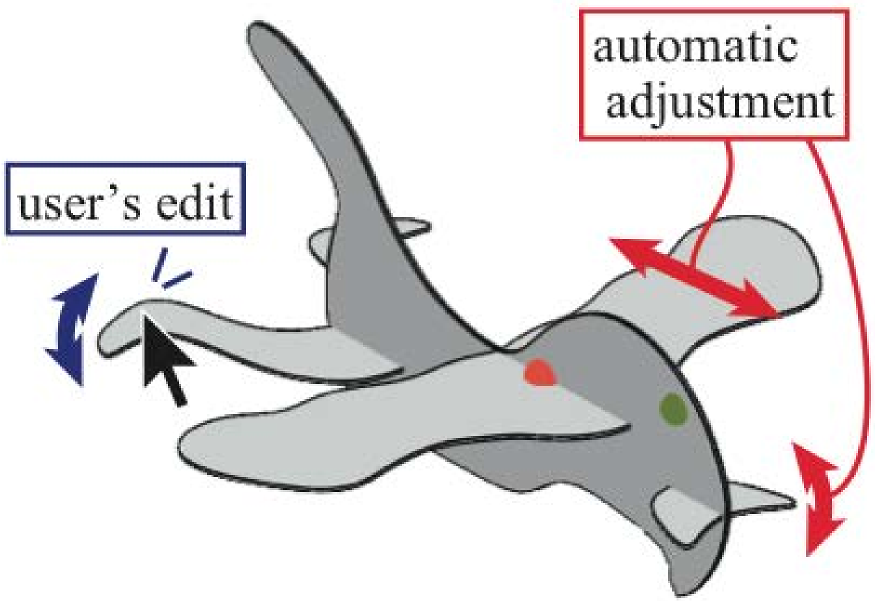
Umetani, Koyama, Schmidt and Igarashi, 2014

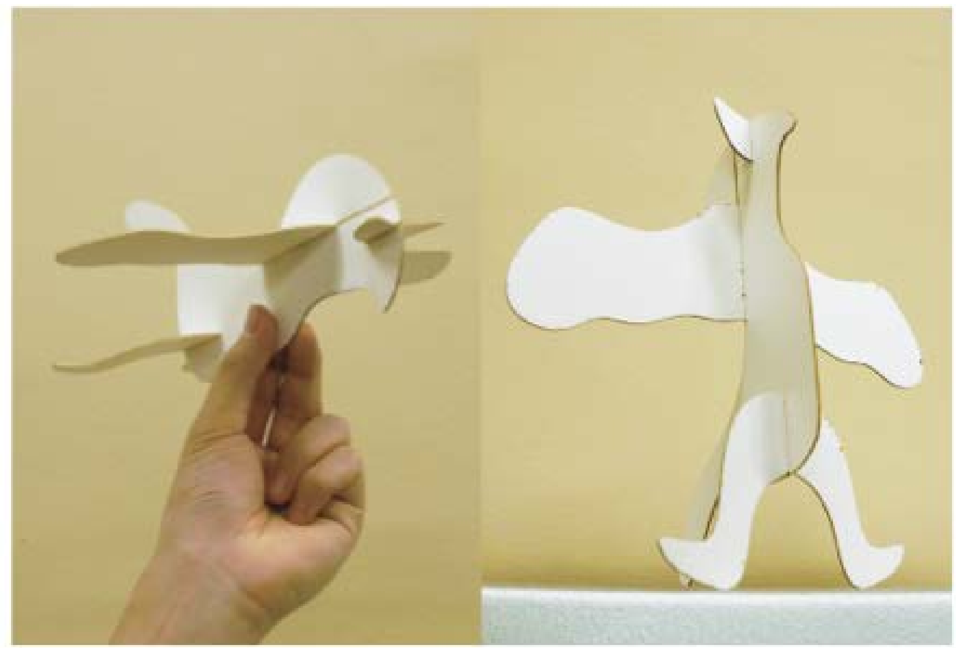
Designing for Mechanical Function



Umetani, Koyama, Schmidt and Igarashi, 2014

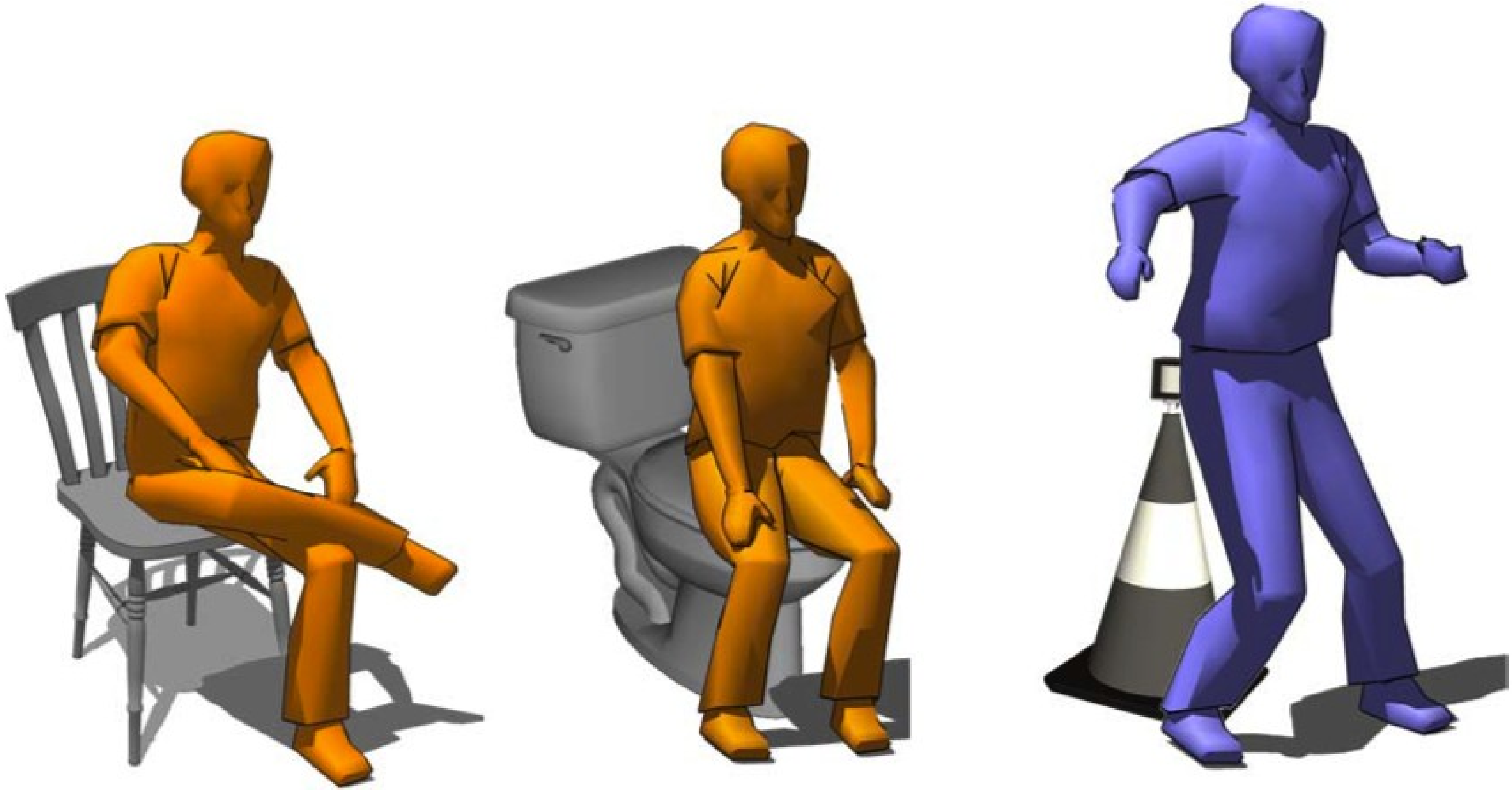
Designing for Mechanical Function





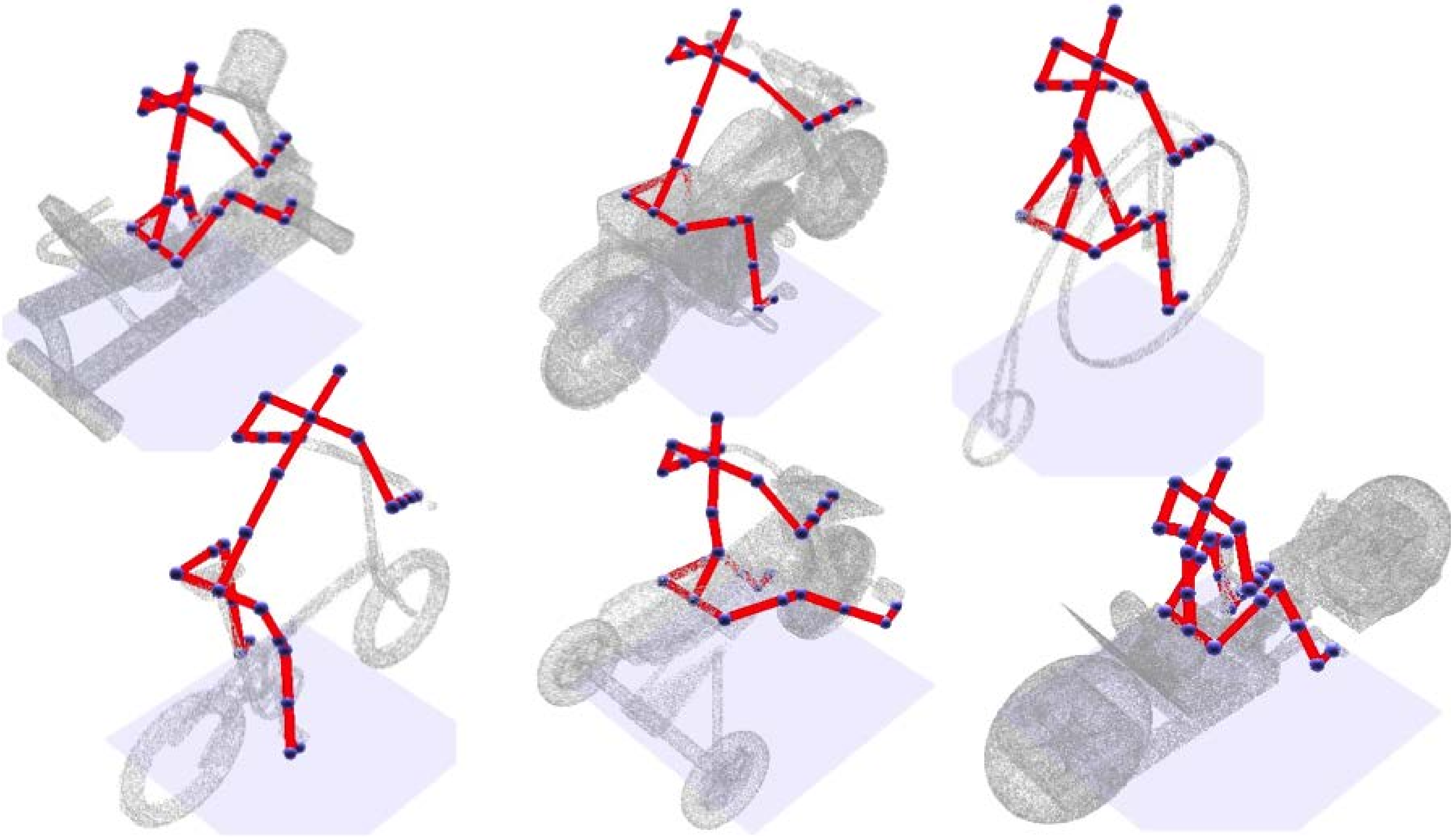
Umetani, Koyama, Schmidt and Igarashi, 2014

What makes a chair a chair?



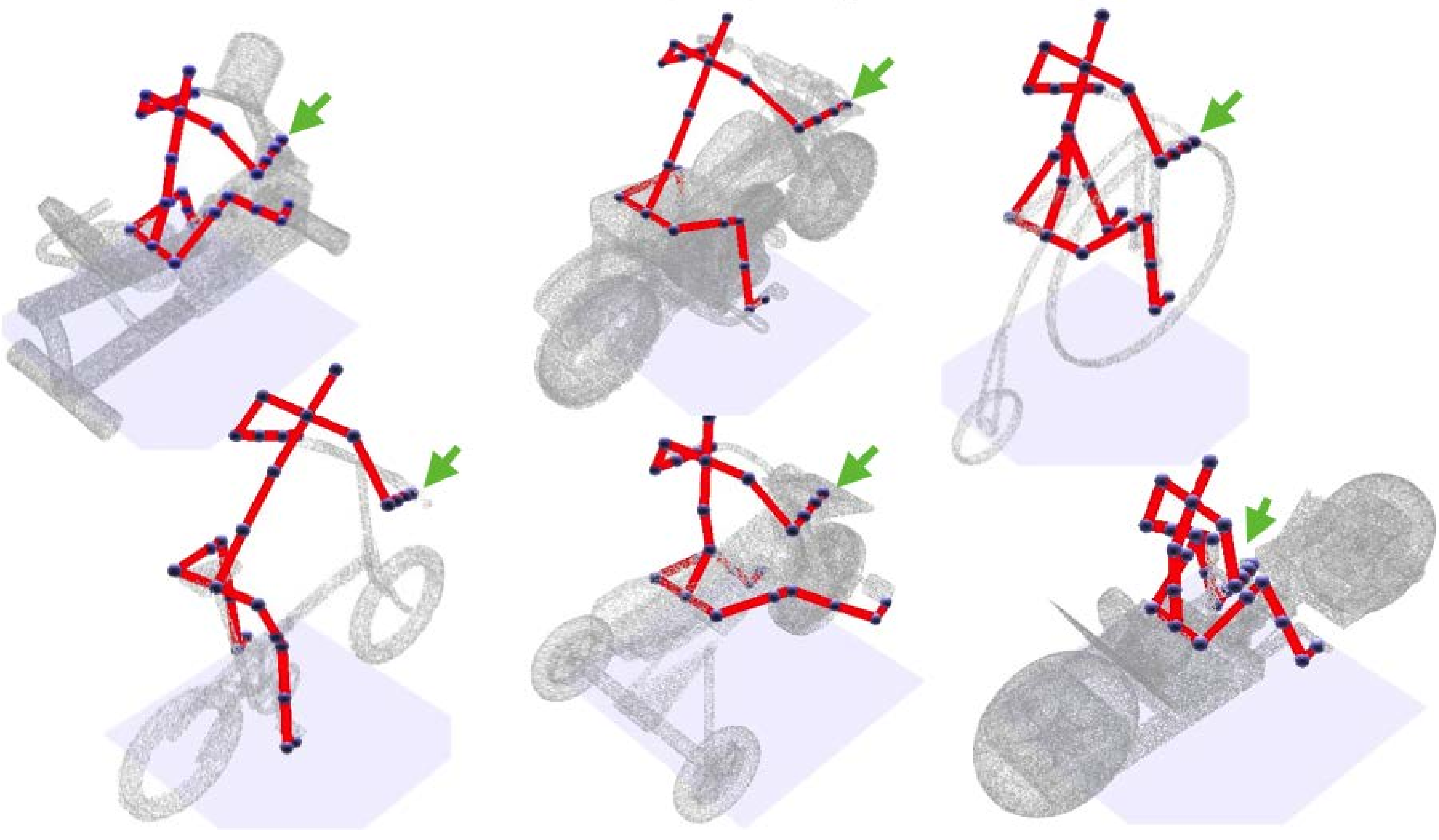
Grabner, Gall and Van Gool, 2011

Human-Centric Shape Analysis



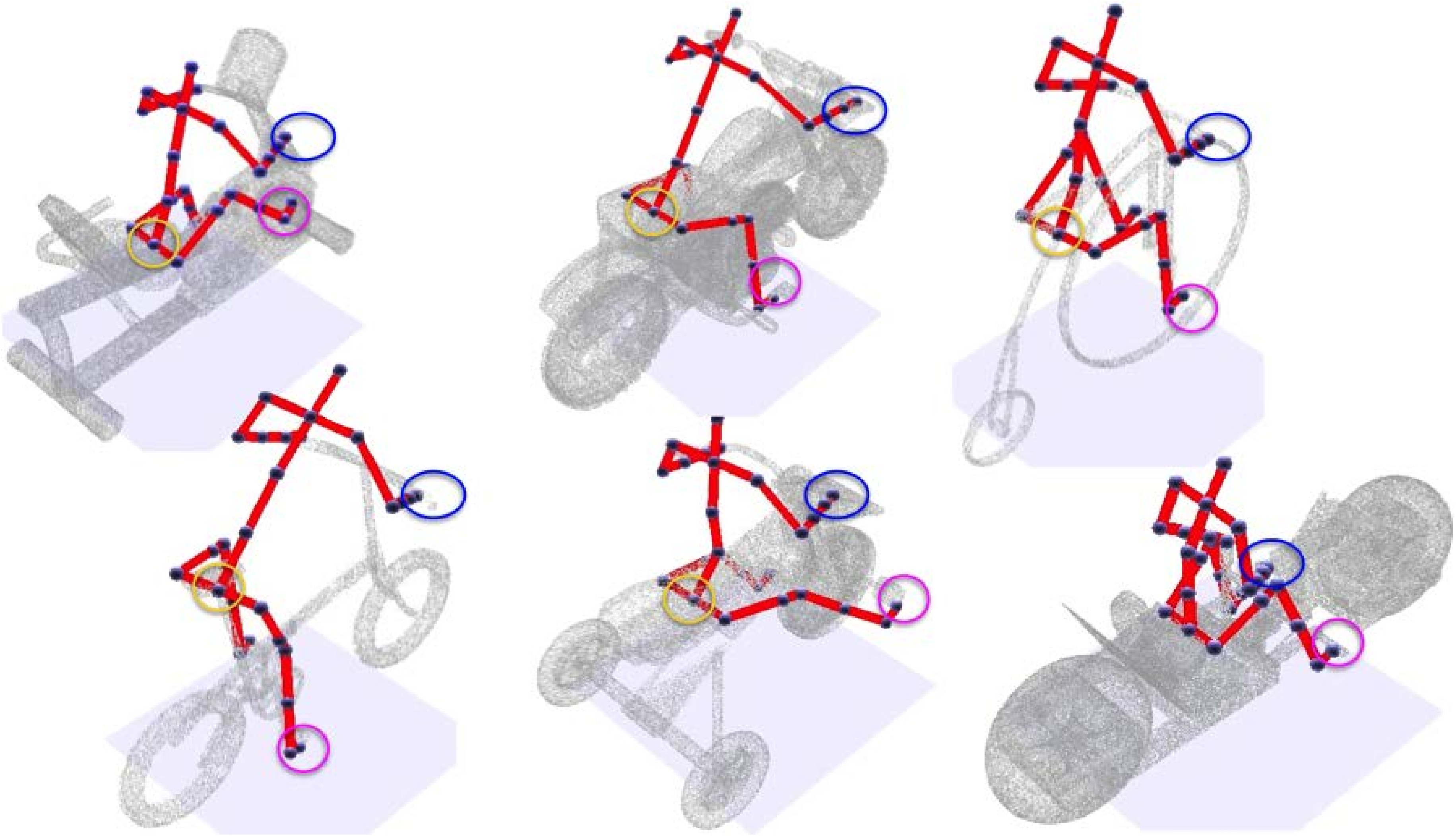
Kim, Chaudhuri, Guibas and Funkhouser, 2014

Point-to-Point Correspondences



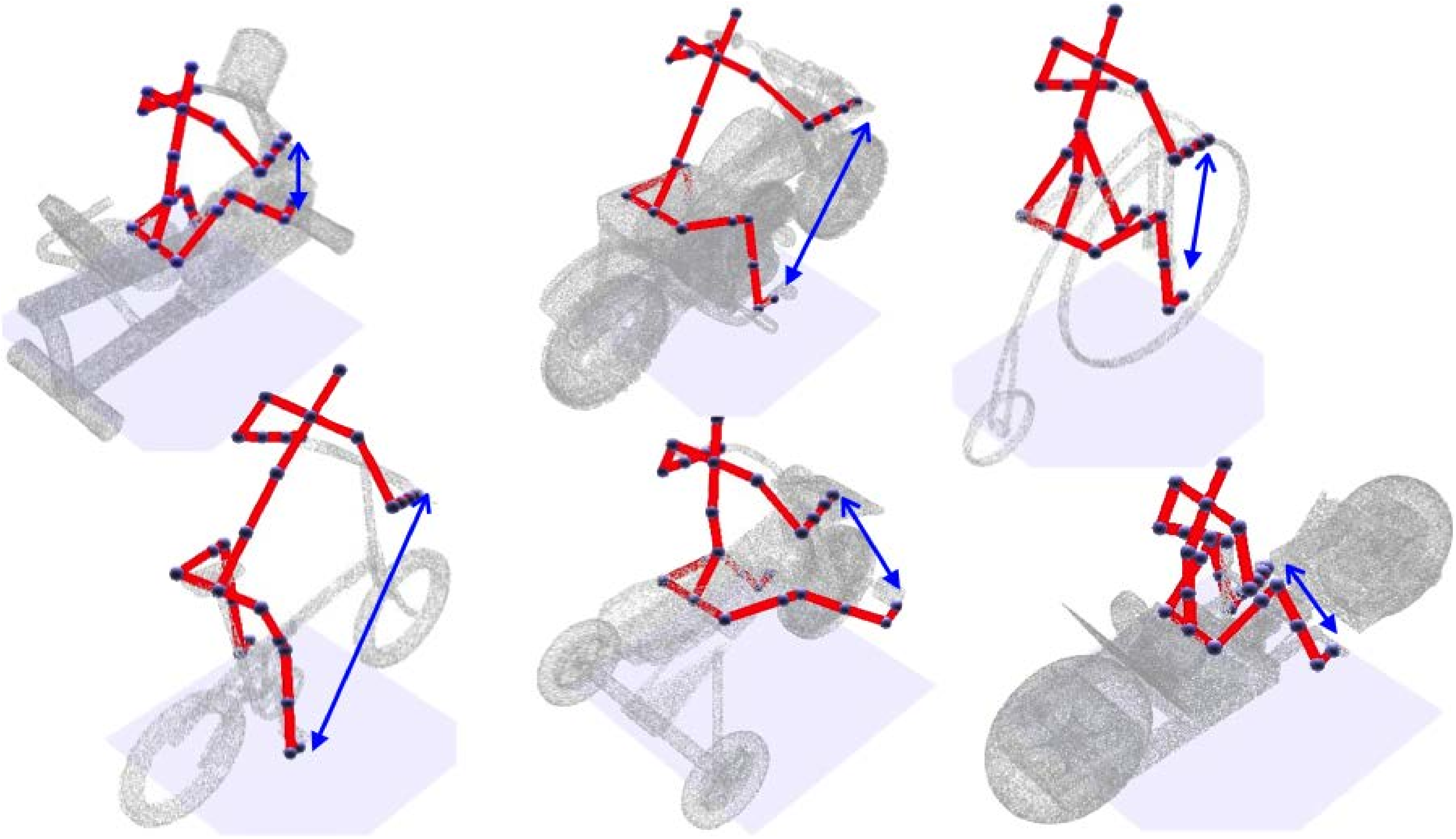
Kim, Chaudhuri, Guibas and Funkhouser, 2014

Functional Parts



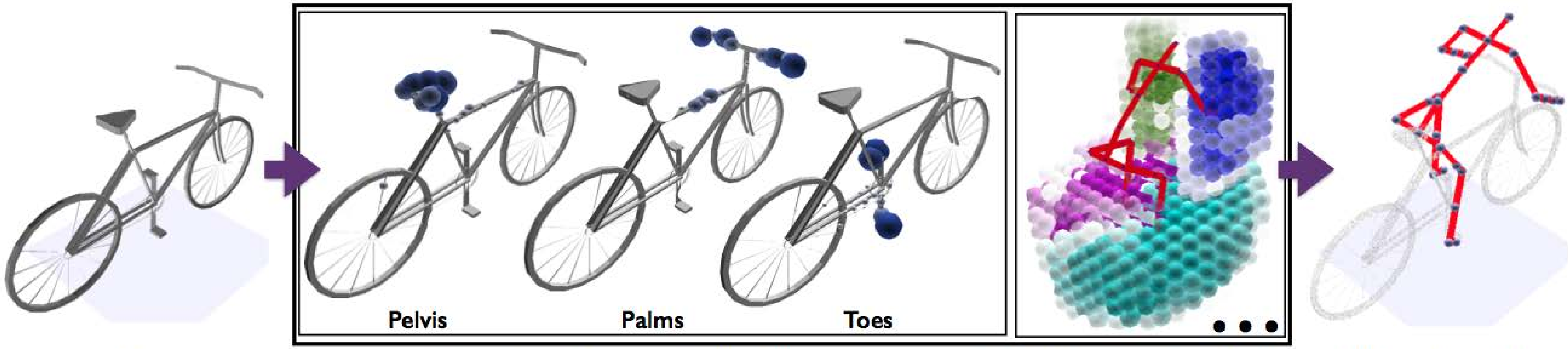
Kim, Chaudhuri, Guibas and Funkhouser, 2014

Structural Variations

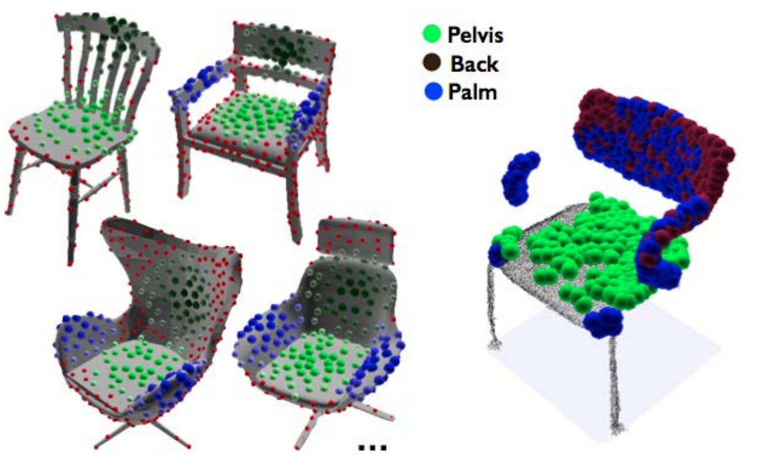


Kim, Chaudhuri, Guibas and Funkhouser, 2014

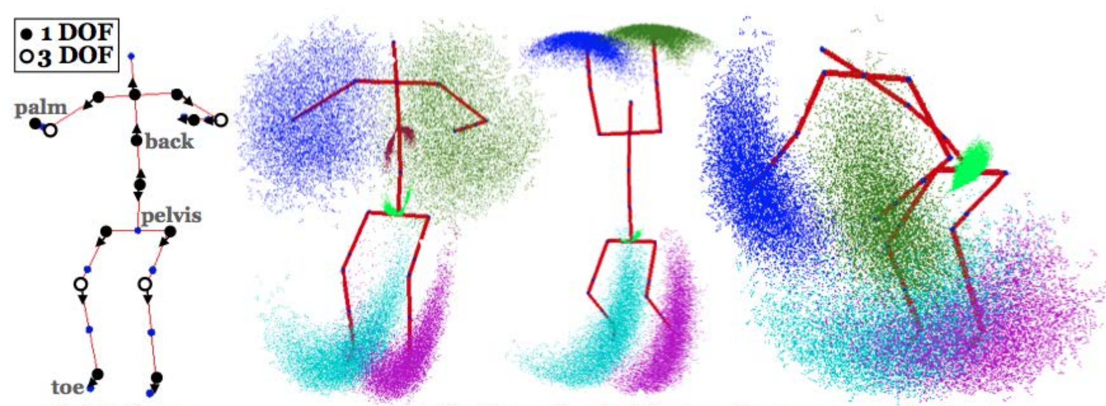
Pose Prediction Pipeline



(a) Input Shape (b) Predicted Contact Probabilities (c) End Effector Probabilities (d) Predicted Pose



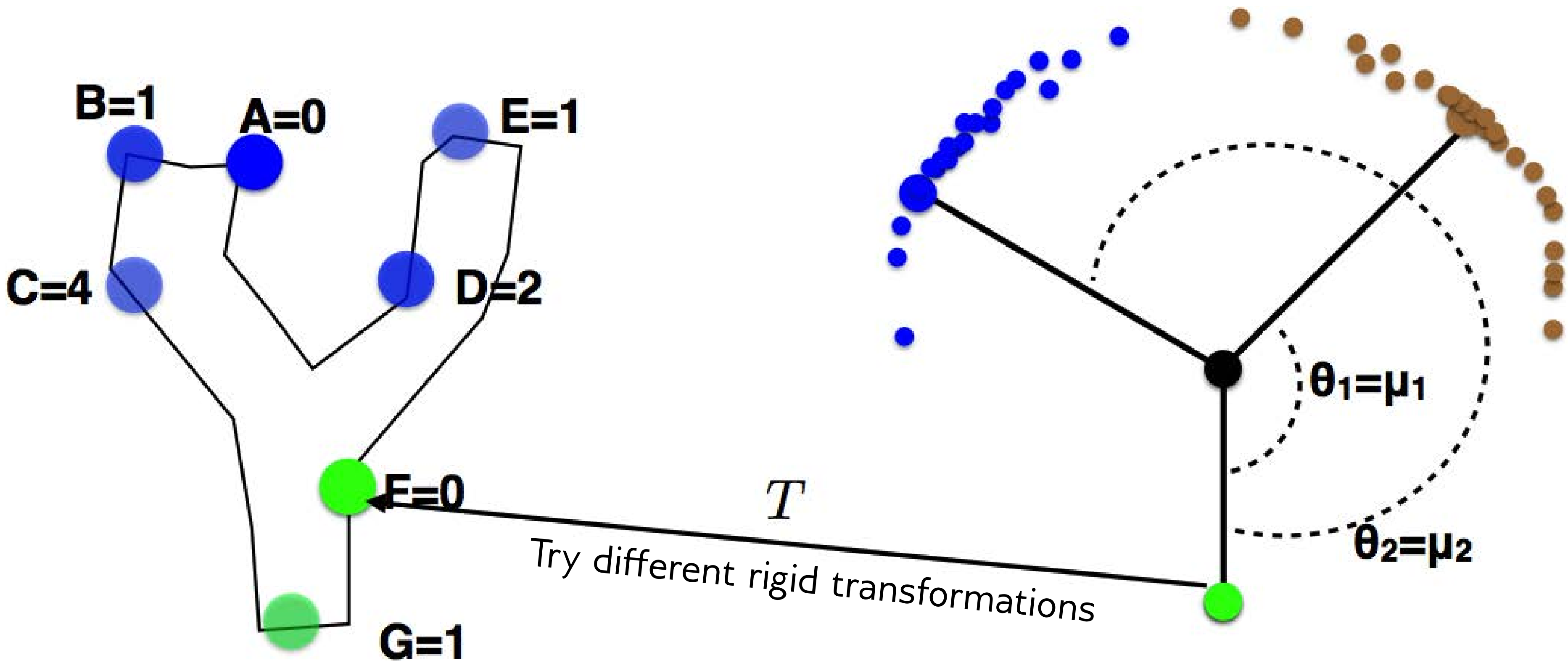
(a) Training Examples (b) High-probability contacts



Joint Types Distribution of end effectors for some sampled poses

Kim, Chaudhuri, Guibas and Funkhouser, SIGGRAPH 2014

Key to efficient optimization:
Sample pose prior and contact priors independently

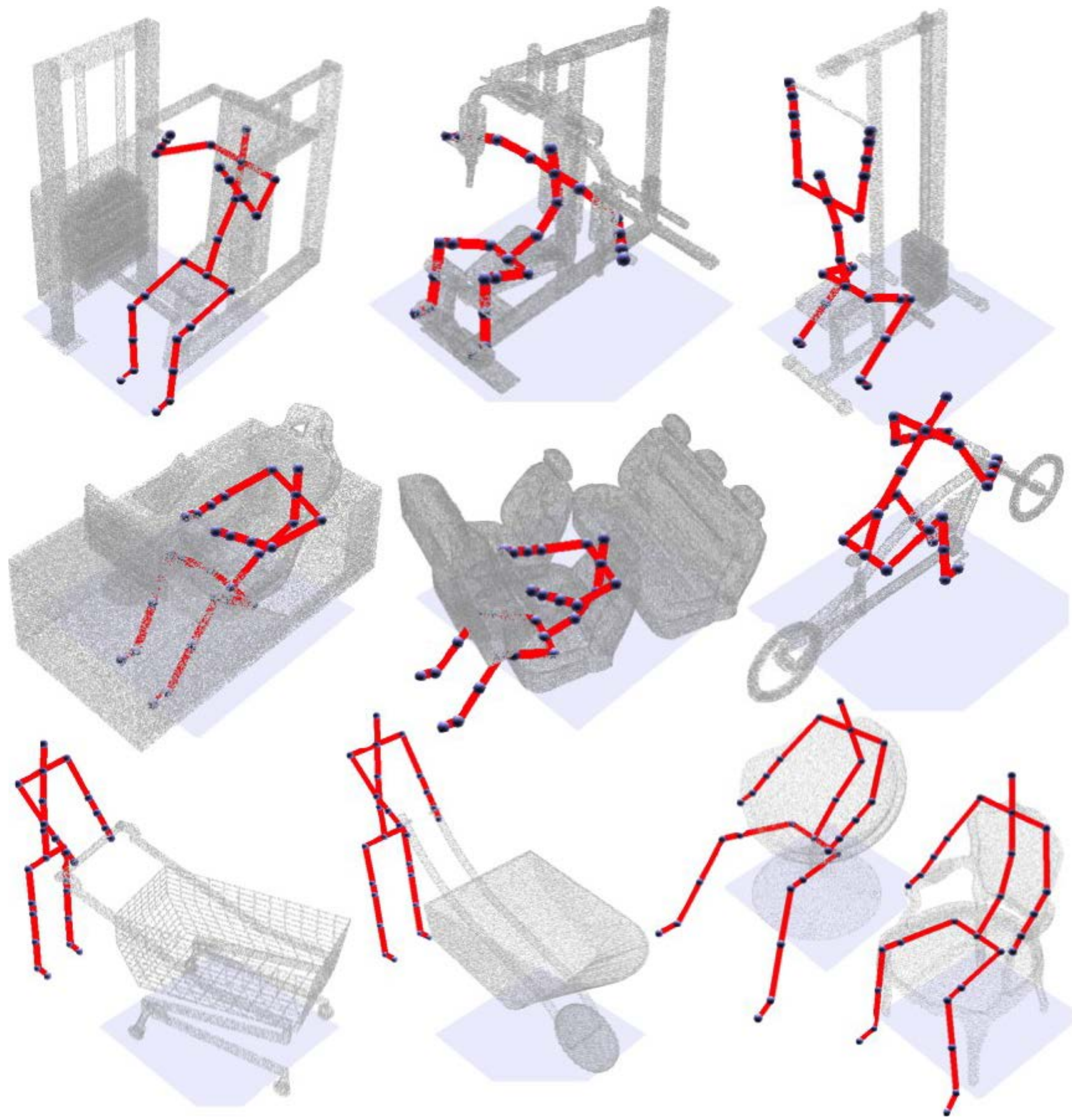


Contact distribution

End-effector distribution

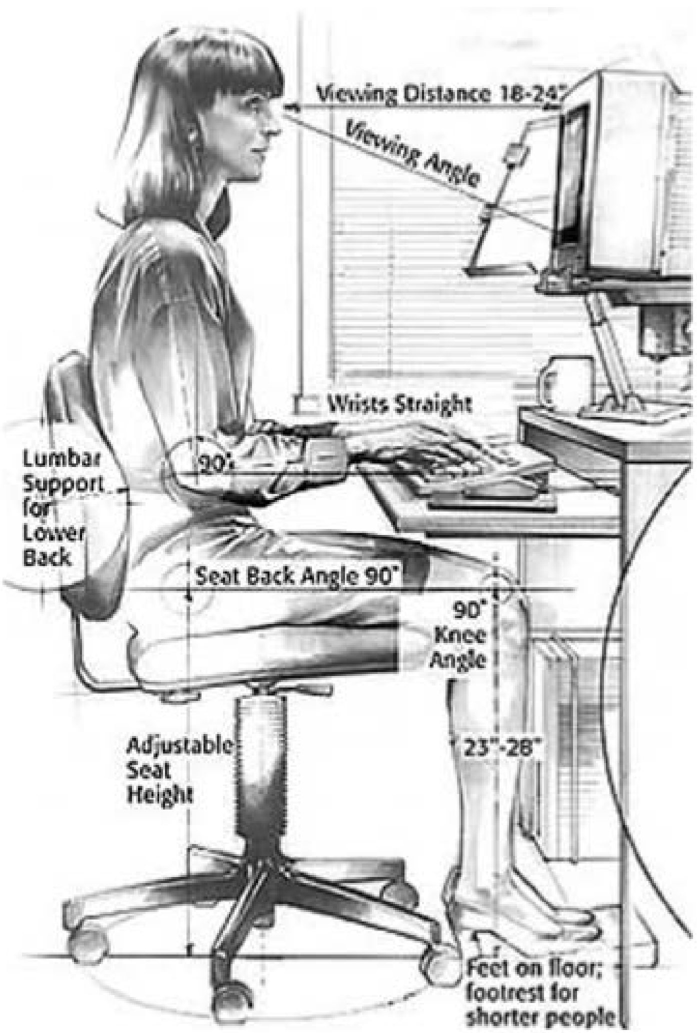
Kim, Chaudhuri, Guibas and Funkhouser, SIGGRAPH 2014

Learning to Predict Human Interaction



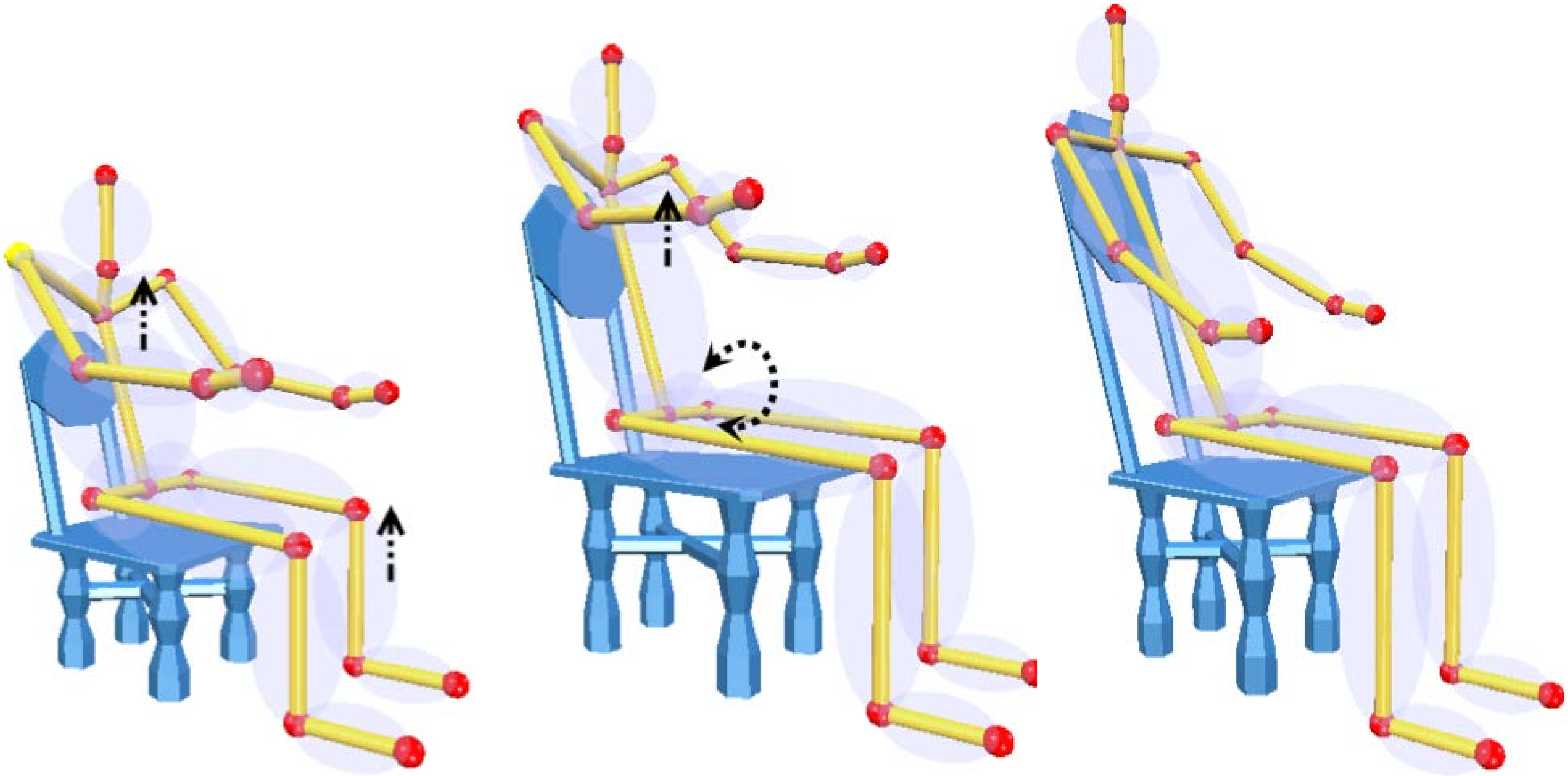
Kim, Chaudhuri, Guibas and Funkhouser, SIGGRAPH 2014

Designing for Human Interaction



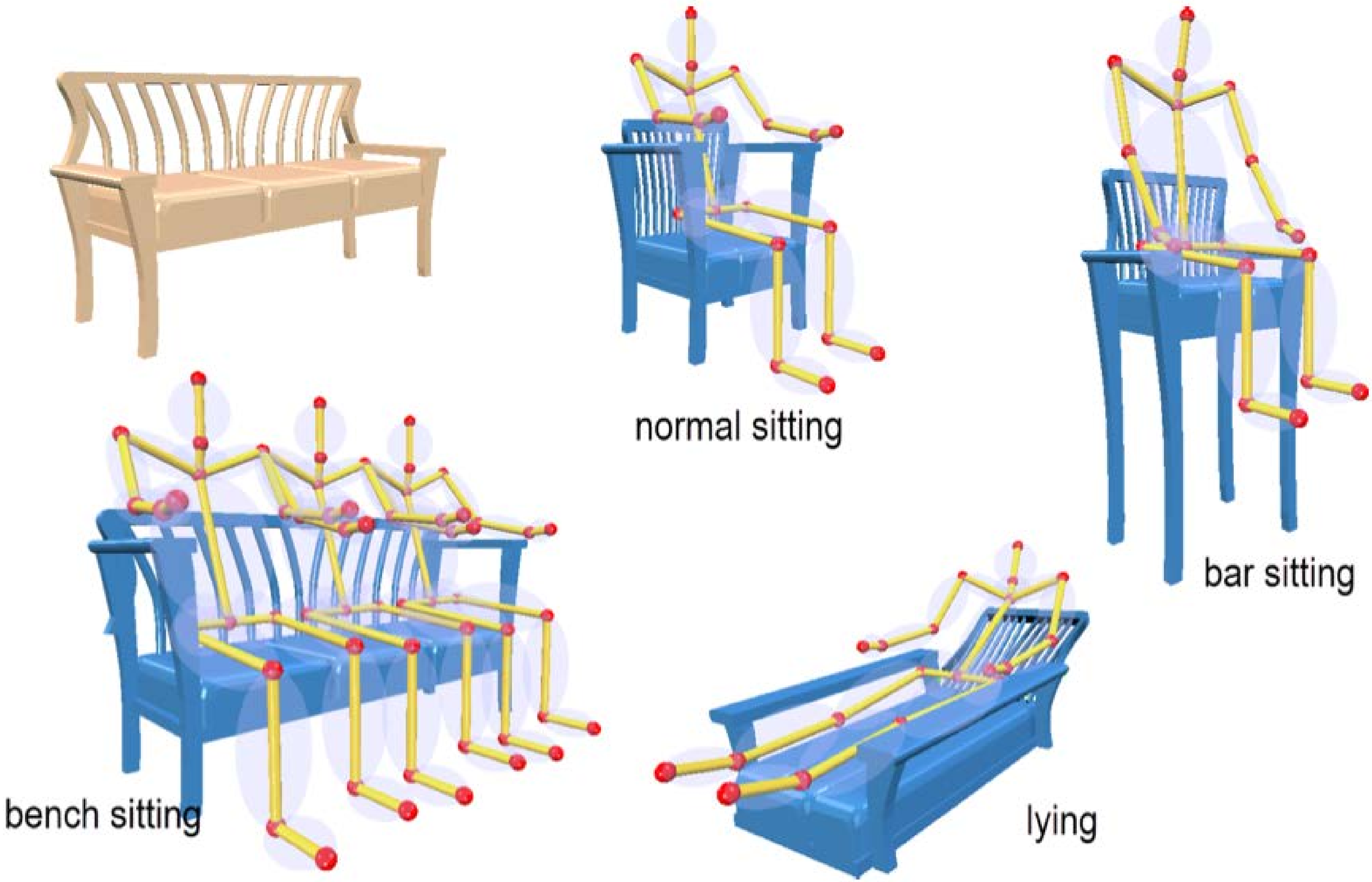
Zheng, Dorsey and Mitra, 2014

Shape Adjustment for Body Type



Zheng, Dorsey and Mitra, 2014

Shape Adjustment for Body Pose



Zheng, Dorsey and Mitra, 2014

Summary

- **"High-level" geometric analysis**
- **Probabilistic models** can characterize the structure of "plausible" objects, and generate new ones
- Design intent can be captured through **semantic attributes**, **mechanical function** and **human interaction**
- Models of structure, attributes, function and interaction can be automatically learned from **(big) data**